

SUNNER A LEVEL FURTHER MATHS

STUDENT NAME:





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About the Summer Work

A Level Further Mathematics takes as a base your knowledge of topics from GCSE and extends these into new areas of mathematics such as complex numbers.

You need to ensure that when you start in September that you are confident on all the grade 8/9 content covered at GCSE. You will have an initial assessment in your first week covering some of the grade 8/9 content covered at GCSE.

Guidance

- Complete these tasks on A4 paper and bring them with you to your first further maths lesson.
- Each page should be labelled with the title of the task and question numbers included.
- Work should be self-marked using green pen and corrected for errors.
- Attempt every question and always show your working.
- Spend additional time on tasks you struggle with, using corbettmaths videos to help you where necessary.
- This booklet also contains significant additional information. We would encourage you to complete all the tasks including the optional ones to fully prepare for Sixth Form study.
- Use the week-by-week schedule as a guide to how much you should be aiming to complete each week.



Welcome to Further Mathematics

Subject outline

Further Mathematics has seen the largest % increase in the number of students being entered for both the AS and A-Level exams, an increase of 9.6% for A-Level Further Mathematics. The subject sharpens many key skills, including the ability to get to grips with problems, something that lies at the centre of many fields. Students who study Further Mathematics at A-Level relish a challenge and enjoy investigating different processes. Further Mathematics students will have the ability to create an interpret mathematical models of real-world phenomena, whilst also having a mastery of a wide range of mathematical techniques and methods. Outside of lesson time, students need to use their independent study time to practice maths and check through solutions from a range of resources including online retrieval practice exercises, textbook procedural practice, and past exam papers.

Further Mathematics has always been a highly valued A-Level by Universities and employers due to its complex content and the demands of the course. Further Mathematics opens pathways for students to a wide range of courses that require students to be highly numerate and strong logical thinkers. In our technology focused society, further mathematics students can often show innovation and creativity in approaching a challenge and working to find a solution, traits which are essential in the modern-day work force. This has motivated our decision to focus our optional modules on Decision Mathematics which lends itself to a more technological society.

Students will study the Edexcel Specification for A Level Further Mathematics, with students sitting the AS Level at the end of Year 1 and the full A-Level at the end of Year 2.

AS- Level:

Paper 1 (50% of the AS Level) – Core Pure Mathematics (concepts such as complex numbers, vectors, matrices, proof and calculus)

Paper 2 (50% of the AS Level) – Decision Mathematics 1 and Decision Mathematics 2 (concepts such as algorithms, Route Inspection, Game Theory, Linear Programming and Critical Path Analysis)



A Level:

Paper 1 and 2 (50% of the A Level) – Core Pure Mathematics (concepts such as complex numbers, vectors, matrices, proof, calculus, polar coordinates, hyperbolic functions)

Paper 3 (25% of the A Level) – Decision Mathematics 1 (concepts such as algorithms, Route Inspection, Linear Programming and Critical Path Analysis)

Paper 4 (25% of the A Level) – Decision Mathematics 2 (concepts such as Game Theory, Dynamic Programming, Decision Trees, Recurrence Relations and Network Flows)



Careers & Higher Education

Studying A Level Further Mathematics opens a wide array of career and higher education opportunities due to the advanced analytical and problem-solving skills acquired. A Level Further Mathematics is a facilitating subject, meaning that is a highly respected A Level qualification. If you are interested in studying engineering, economics, mathematics, physics, statistics, actuarial science or computer science A Level Further Mathematics is recommended. If you are interested in other routes such as biochemistry, dentistry, business studies, geography or accounting some universities may list mathematics as a useful subject but not essential.

Maths careers are some of the most highly paid careers available. Research shows that on average A Level maths students earn 11% more than other students during their lifetime. Many believe that taking maths at university has limited fields since it doesn't move straight into a vocation. However, this is certainly not the case. Students who continue maths at university can move into various careers, from graduate roles within the finance industry to working in a graduate role within the civil service. Engineering has many different degree routes and courses and is one of the most popular areas that students choose to work in after university.

An example of a highly mathematical career is a data scientist. A data scientist uses statistical and computational methods to analyse large datasets, extract insights, and inform decision-making in various industries, including technology, finance, healthcare, and marketing. This role involves working with complex data, building predictive models, and communicating findings to stakeholders. A significant part of a data scientist's job is to communicate complex technical finding to non-technical stakeholders. The ability to explain mathematical concepts clearly and concisely, homed in Further Mathematics, is crucial. In addition, the role can often involve using programming languages and software tools to handle and analyse data.

Links to key information:

dixons6a.com/uploads/files/Maths.pdf

https://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2017/sp ecification-and-sample-assesment/a-level-I3-further-mathematics-specification.pdf https://amsp.org.uk/teachers/11-16-maths/transition-to-level-3-maths/where-mathsmeets-the-world-of-work/



Summer work tasks Task 1

There are 3 practice papers to complete, with answers, that are all revision of key grade 8/9 topics from GCSE. These topics are essential to the study of A Level Further Mathematics and students need to ensure they fully understand each concept. **Students will be assessed in the first lesson on these topics** to ensure they are starting the course with a grounded understanding of algebra from GCSE. Please see the guidance on the previous page on how to set out your work. Below is a suggested week-by-week schedule to help you organize your time.

Task 2

There are several problems for you to attempt to prepare you for the problem-solving skills that will be necessary when studying A-Level Further Mathematics. Attempt all the questions to improve your problem-solving skills.

Week	Exercise
1	Paper 1 Grade 9 Questions
2	Paper 2 Grade 9 Questions
3	Paper 3 Grade 9 Questions
4	Problems 1, 2 and 3
5	Problems 4, 5 and 6
6	Problems 7 and 8



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Reading list

Suggested reading:

The Codebook by Simon Singh

The Simpsons and Their Mathematical Secrets by Simon Singh

Infinity: The Quest to Think the Unthinkable by Brian Clegg

The Man who knew Infinity by Robert Kanigel

Humble Pi: A Comedy of Maths Errors by Matt Parker

Suggested viewing:

bbc.co.uk/iplayer/episode/b0074rxx/horizon-19951996-fermats-last-theorem



Please check the examination details below befo	re entering your candidate information
Candidate surname	Other names
Centre Number Candidate Number	evel 2 GCSE (9-1)
Aiming for Grade 9	
Pap refe	er 1MA1/1H
Mathematics	•
PAPER 1 (Non-Calculator) Higher Tier	
42 marks 40 minutes	
You must have: Ruler graduated in centime protractor, pair of compasses, pen, HB penc Formulae Sheet (enclosed). Tracing paper m	il, eraser,

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided - there may be more space than you need.
- You must **show all your working**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- Calculators may not be used.

Information

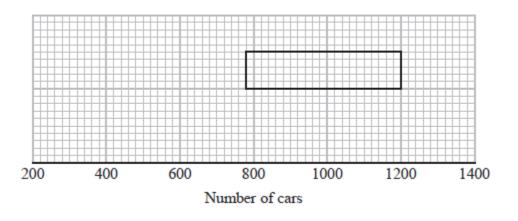
- The total mark for this paper is 42. There are 10 questions.
- Questions have been broadly arranged in an ascending order of mean difficulty, as found by students achieving Grade 9 in the Summer and November 2023 examinations.
- Questions marked with an asterisk (*) also appear on the Foundation Tier paper.
- The marks for each question are shown in brackets
 use this as a guide as to how much time to spend on each question.



Answer all questions. Write your answers in the spaces provided. You must write down all the stages in your working.

Alice recorded the number of cars going into a village on each of 80 days.
 The incomplete table and the incomplete box plot give information about her results.

	Number of cars
Least number	300
Lower quartile	
Median	900
Upper quartile	
Range	1000



- (a) (i) Use the information in the table to complete the box plot.
 - (ii) Use the information in the box plot to complete the table.

(3)

- On some of these 80 days Alice saw fewer than 1200 cars going into the village.
- (*b*) Work out an estimate for the number of days Alice saw fewer than 1200 cars going into the village.



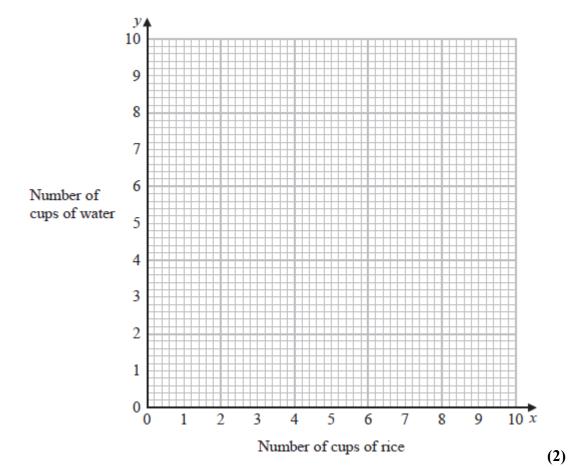
(2) (Total for Question 1 is 5 marks)

.....

2 To cook rice

the number of cups of rice (x): the number of cups of water (y) = 4:5

(*a*) Use this information to draw a graph to show the relationship between the number of cups of rice and the number of cups of water needed to cook rice.



(b) (i) Find the gradient of the line drawn in part (a).

(ii) Explain what this gradient represents.

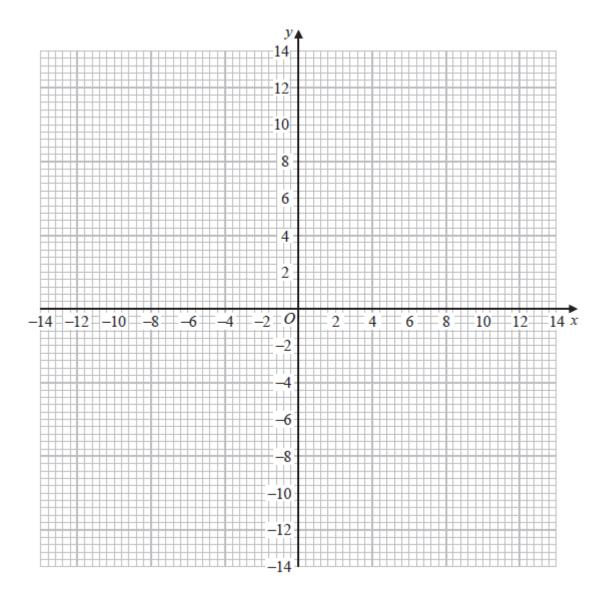


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(1) (Total for Question 2 is 4 marks)

3 (a) On the grid, draw the graph of $x^2 + y^2 = 169$



(2)

(3)

(b) Use your graph to find estimates for the solutions of the simultaneous equations

$$x^2 + y^2 = 169$$
$$2y = 3x$$

.....

(Total for Question 3 is 5 marks)



 4 7 kg of carrots and 5 kg of tomatoes cost a total of 480p cost of 1 kg of carrots : cost of 1 kg of tomatoes = 5 : 9
 Work out the cost of 1 kg of carrots and the cost of 1 kg of tomatoes.

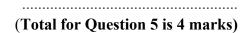
carrotsp

tomatoesp (Total for Question 4 is 4 marks)

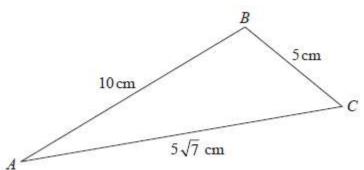
5 Write
$$\frac{3\sqrt{3}}{4-\sqrt{3}} - \frac{2}{\sqrt{3}}$$
 in the form $\frac{a\sqrt{3}+b}{c}$ where *a*, *b* and *c* are integers.



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6 Here is triangle *ABC*.



Find the size of angle *ABC*. You must show all your working.

> • (Total for Question 6 is 4 marks)

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Solid A and solid B are similar.
The ratio of the height of solid A to the height of solid B is 2 : 5 The volume of solid A is 12 cm³ Work out the volume of solid B.

(Total for Question 7 is 3 marks)

8 The 2nd term of a geometric sequence is $3 + 2\sqrt{2}$ The 3rd term of the sequence is $13 + 9\sqrt{2}$ Find the value of the common ratio of the sequence. Give your answer in the form $a + \sqrt{b}$ where *a* and *b* are integers. You must show all your working.



..... (Total for Question 8 is 4 marks)

Find the set of possible values of *x* for which 9 $4x^2 - 25 < 0$ You must show all your working.

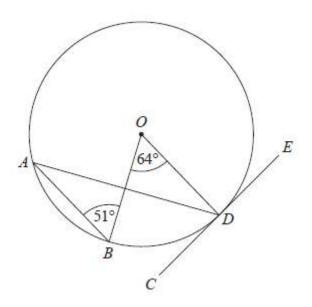
and $12 - 5x - 3x^2 > 0$

..... (Total for Question 9 is 5 marks)



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10 *A*, *B* and *D* are points on a circle with centre *O*. *CDE* is the tangent to the circle at *D*.



Work out the size of angle *ADC*. Write down any circle theorems you use.



• (Total for Question 10 is 4 marks)

TOTAL FOR PAPER IS 42 MARKS

Candidate surname	C	Other names	
Centre Number Candidate N Candidate N Candidate N Candidate N Candidate N Candidate N Candidate N		l 2 GCSE (9–1)	
Aiming for Grade 9		114 - 273 -	
	Paper reference	1MA1/2F	
Mathematics PAPER 2 (Calculator) Higher Tier			
40 marks 40 minutes You must have: Ruler graduated in c			ark
protractor, pair of compasses, pen, H Formulae Sheet (enclosed). Tracing p	the second s		

Instructions

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- Answer **all** questions.
- Answer the questions in the spaces provided there may be more space than you need.
- You must **show all your working**.
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.

Information

- The total mark for this paper is 40. There are 12 questions.
- Questions have been broadly arranged in an ascending order of mean difficulty, as found by students achieving Grade 9 in the Summer and November 2023 examinations.



- Questions marked with an asterisk (*) also appear on the Higher Tier paper.
- The marks for **each** question are shown in brackets

– use this as a guide as to how much time to spend on each question.

Answer all questions.

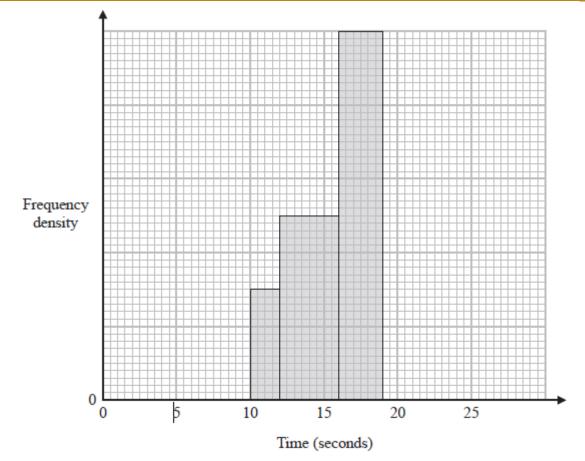
Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The incomplete table and the incomplete histogram give information about the times taken by some students to run a race.

Time (<i>t</i> seconds)	Frequency
$10 < t \le 12$	
$12 < t \le 16$	10
$16 < t \le 19$	15
$19 < t \le 21$	9
$21 < t \le 26$	7





None of these students had a time for the race such that t < 10 or t > 26 (*a*) Use the histogram to complete the table.

(1)

(2)

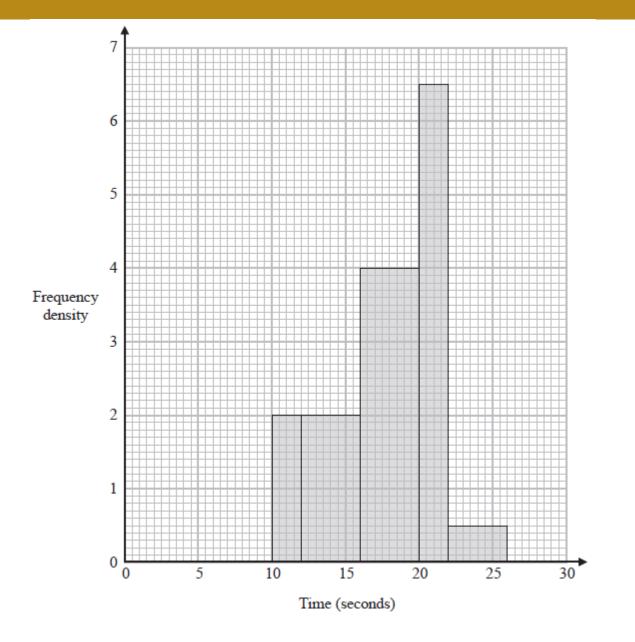
(*b*) Use the table to complete the histogram.

The histogram below gives information about the times taken by 43 students to run a different race.

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(c) Work out an estimate for the median of the times taken by these 43 students to run the race.

..... seconds (3) (Total for Question 1 is 6 marks)



2 A biased dice is thrown 60 times.

The table shows information about the number that the dice lands on each time.

Number on dice	1	2	3	4	5	6
Frequency	12	7	8	9	9	15

Gethin throws the dice twice.

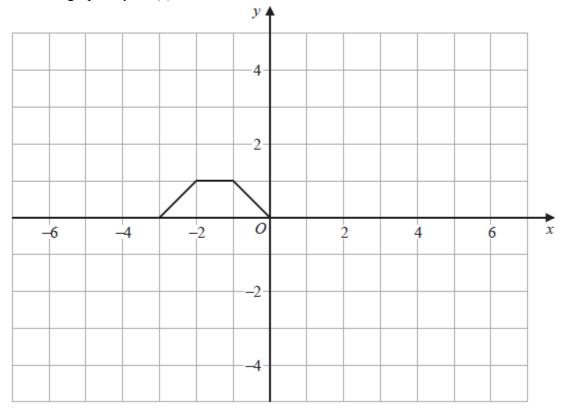
(a) Work out an estimate for the probability that the dice will land on 6 both times.

(3)
Sally is going to throw the same dice *n* times and record the number it lands on each time. She will use her results to work out a more reliable estimate for the probability in part (*a*).
(*b*) What can you say about the value of *n*?
(*c*)
(*f*) (Total for Question 2 is 4 marks)

3



Here is the graph of y = f(x)



On the grid below, draw the graph of y = f(-x)

у 🛉 4 2 \mathbf{x} -2 0 2 4 6 -6 -4 -2 -4



(Total for Question 3 is 1 mark)

*4 The bearing of port *B* from port *A* is 147° Work out the bearing of port *A* from port *B*.

> • (Total for Question 4 is 2 marks)

5 2a: 5c = 6: 254b: 7c = 20: 21

Show that a + b : b + c = 17 : 20



(Total for Question 5 is 3 marks)

6 Write
$$\frac{14}{3x-21} + \left[(x+4) \div \frac{2x^2 - 6x - 56}{2x+3} \right]$$
 in the form $\frac{ax+b}{cx+d}$ where a, b, c and d are integers.

(Total for Question 6 is 4 marks)



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D

7 ABCD is a quadrilateral. E, F, G and H are the midpoints of AB, BC, CD and DA. $\overrightarrow{AH} = \mathbf{a}$ $\overrightarrow{AE} = \mathbf{b}$ $\overrightarrow{DG} = \mathbf{c}$ Prove, using vectors, that EFGH is a parallelogram. A \overrightarrow{A}

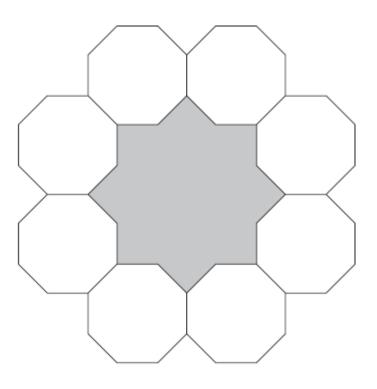
(Total for Question 7 is 4 marks)

8 Show that the equation $x^3 + 2x - 6 = 0$ has a solution between x = 1 and x = 2

(Total for Question 8 is 2 marks)



9 The diagram shows 8 identical regular octagons joined to enclose a shaded shape.



.....

Each octagon has sides of length *a*.

Find, in terms of *a*, an expression for the area of the shaded shape.

Give your answer in the form $p(2+\sqrt{2})a^2$ where p is an integer.

You must show all your working.

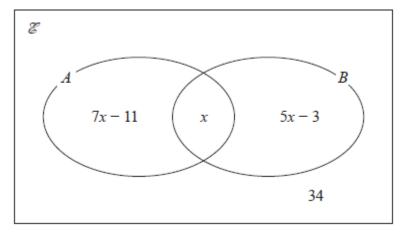


(Total for Question 9 is 5 marks)

10 Vicky has a collection of medals.

The Venn diagram gives information about the number of medals in her collection where

- $\mathscr{E}=\{all medals\}$
- $A = \{ \text{English medals} \}$
- $B = \{ gold medals \}$



Vicky is going to take at random a medal from her collection.

Given that the medal is gold, the probability that the medal is English is $\frac{2}{11}$

Work out the number of medals in Vicky's collection.

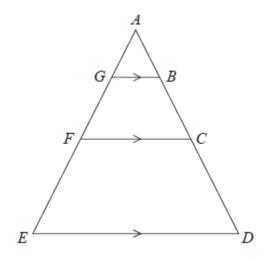
(Total for Question 10 is 4 marks)

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11 Here are three similar triangles, *ABG*, *ACF* and *ADE*.



ABCD and *AGFE* are straight lines. *AB* : *BC* : *CD* = 1 : 2 : 3Show that

area of ABG : area of BCFG : area of CDEF = 1 : 8 : 27

(Total for Question 11 is 3 marks)



12 There are only blue pens and red pens in a box. The number of blue pens is four times the number of red pens. Rita takes at random one pen from the box. She records the colour of the pen and then replaces it in the box. Rita does this *n* times, where n ≥ 2 Write down an expression, in terms of *n*, for the probability that Rita gets a blue pen at least once and a red pen at least once.

(Total for Question 12 is 2 marks)

TOTAL FOR PAPER IS 36 MARKS



Please check the examination details be	elow before entering your candidate information
Candidate surname	Other names
Centre Number Candidate N	Number
Pearson Edexcel Leve	el 1/Level 2 GCSE (9–1)
Aiming for Grade 9	
	Paper reference 1MA1/3H
Mathematics	• •
PAPER 3 (Calculator)	
Higher Tier	
37 marks 40 minutes	
You must have: Ruler graduated in a protractor, pair of compasses, pen, H Formulae Sheet (enclosed). Tracing p	IB pencil, eraser, calculator,

Instructions

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- •
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Answer all questions.

Write your answers in the spaces provided.



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You must write down all the stages in your working.

1 (a) Factorise $a^2 - b^2$

.....

(1)

(b) Show that $2^{40} - 1$ is the product of two consecutive odd numbers.

(2) (Total for Question 1 is 3 marks)



2 A circle has equation $x^2 + y^2 = 25$

The point *P* with coordinates (-3, 4) lies on the circle. Alex says that the tangent to the circle at *P* crosses the *x*-axis at the point (-8, 0)

Is Alex correct?

You must show how you get your answer.

(Total for Question 2 is 4 marks)

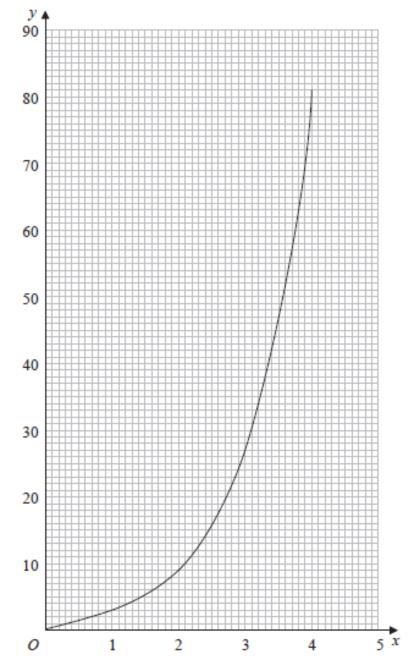
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3 Sana needs to draw the graph of $y = 3^x$ for $0 \le x \le 4$

She draws the graph shown on the grid.



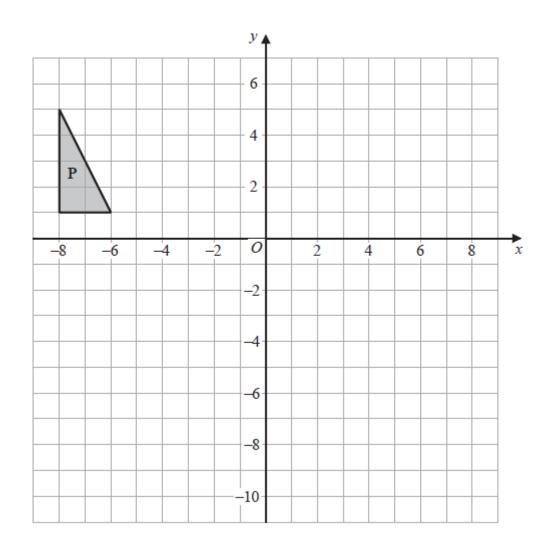
Write down one thing Sana has done wrong.

4



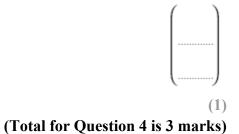
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(Total for Question 3 is 1 mark)



(a) Enlarge triangle **P** by scale factor $-1\frac{1}{2}$ with centre of enlargement (-2, -1) Label your image **Q**.

Triangle P is transformed by a combined transformation of a rotation of 90° anticlockwise about the origin followed by a translation to give triangle R.
Exactly one vertex of triangle P is invariant under the combined transformation.
(b) Find one possible column vector for the translation.



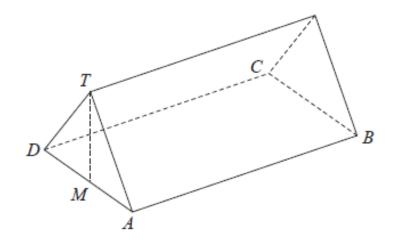
(2)

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5 The diagram shows a triangular prism with a horizontal rectangular base *ABCD*.



M is the midpoint of *AD*.

The vertex T of the prism is vertically above M. AB = 14.7 cm BC = 3.8 cm MT = 2.3 cm

P is the point on *AB* such that

AP : PB = 5 : 2

Calculate the size of the angle between *TP* and the base *ABCD* of the prism. Give your answer correct to 1 decimal place.

• (Total for Question 5 is 4 marks)



6 An expression for the *n*th term of the sequence of triangular numbers is $\frac{n(n+1)}{2}$

Prove that the sum of any two consecutive triangular numbers is a square number.

(Total for Question 6 is 3 marks)

The floor plan of a house is drawn using a scale of 1 : 50
 On the plan, a room in the house has a floor area of 48 cm²
 Work out the real area of the floor of this room.
 Give your answer in m²

..... m²

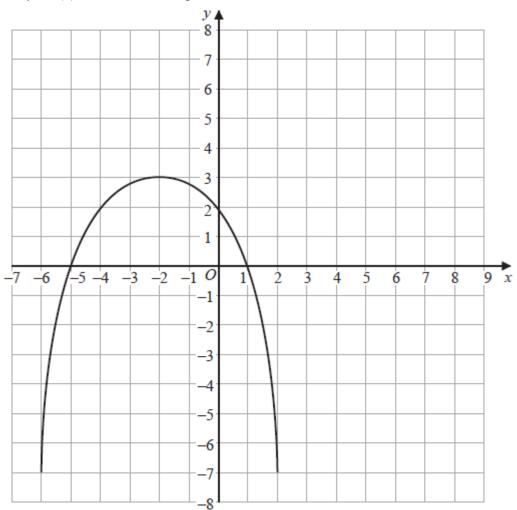
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(Total for Question 7 is 3 marks)

8 The graph of y = f(x) is shown on the grid.



On the grid, sketch the graph of y = f(-x) + 3

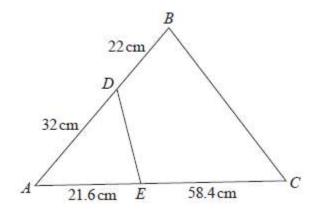
(Total for Question 8 is 2 marks)

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9 The diagram shows triangle *ABC* and triangle *AED*.



Show that triangle *ABC* and triangle *AED* are similar.

(Total for Question 9 is 2 marks)



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10 There is a total of *y* counters in a box.

There are *x* pink counters and 5 blue counters in the box.

The rest of the counters are green.

x: y = 1:3

Freda takes at random two counters from the box.

Find, in terms of x, an expression for the probability that Freda takes two counters of the same colour.

Give your answer as a fraction in the form $\frac{ax^2 + bx + c}{dx^2 + ex}$ where *a*, *b*, *c*, *d* and *e* are integers.

(Total for Question 10 is 5 marks)



Ebony makes some bracelets to sell.The materials to make all the bracelets cost £190, correct to the nearest £5

Ebony sells all the bracelets for a total of £875, correct to the nearest £5 The total time taken to make and sell all these bracelets was 72 hours, correct to the nearest hour.

Ebony uses this method to calculate her hourly rate of pay

Hourly rate of pay = $\frac{\text{total selling price} - \text{total cost of materials}}{\text{total time taken}}$

The minimum hourly rate of pay for someone of Ebony's age is £8.20

By considering bounds, determine if Ebony's hourly rate of pay was definitely more than £8.20

You must show all your working.

(Total for Question 11 is 4 marks)

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12 Given that the vector
$$a \begin{pmatrix} 2 \\ 6 \end{pmatrix} + b \begin{pmatrix} 8 \\ 2 \end{pmatrix}$$
 is parallel to the vector $\begin{pmatrix} 13 \\ 6 \end{pmatrix}$

find an expression for *b* in terms of *a*.

(Total for Question 12 is 3 marks)

TOTAL FOR PAPER IS 37 MARKS

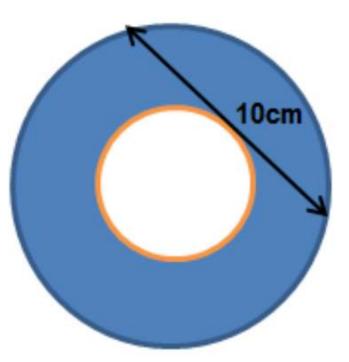


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Task 2

Problem 1:

Can you work out the shaded area in the diagram (the line shown just touches the smaller circle)?



Problem 2:

Find the value of

$$\frac{99}{100} \times \frac{80}{81} \times \frac{63}{64} \times \frac{48}{49} \times \frac{35}{36} \times \frac{24}{25} \times \frac{15}{16} \times \frac{8}{9} \times \frac{3}{4}.$$

Write your answer in the form $\frac{a}{b}$, where *a* and *b* are positive integers with no common factors other than 1.



Problem 3:

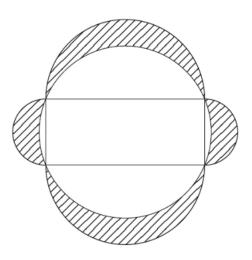
A point *E* lies outside the rectangle ABCD such that CBE is an equilateral triangle. The area of the pentagon ABECD is five times the area of the triangle CBE.

What is the ratio of the lengths *AB* : *AD*?

Write your answer in the form a : 1.

Problem 4:

Four semicircles are drawn on the sides of a rectangle with width 10 cm and length 24 cm. A circle is drawn that passes through the four vertices of the rectangle.



What is the value, in cm^2 , of the shaded area?

Problem 5:



The points A(1,2) and B(-2,1) are two vertices of a rectangle ABCD. The diagonal CA produced passes through the point (2,9). Calculate the coordinates of the vertices C and D.

Problem 6:

- (a) Which positive integer in the range from 1 to 250 has more different prime divisors than any other integer in this range? [3 marks]
- (b) When n = 5 the product n(n + 1)(n + 2) can be written as the product of four distinct primes. Indeed, when n = 5

$$n(n+1)(n+2) = 5 \times 6 \times 7 = 2 \times 3 \times 5 \times 7.$$

What is the least positive integer *n* such that n(n + 1)(n + 2) can be written as a product of *five* distinct primes? [3 marks]

Problem 7:

Powerful quadratics

Problem

(i) Find all real solutions of the equation

$$(x^2 - 7x + 11)^{(x^2 - 11x + 30)} = 1.$$

(ii) Find all real solutions of the equation

$$(2 - x^2)^{(x^2 - 3\sqrt{2}x + 4)} = 1.$$

Problem 8:



Scary sum

Problem

Evaluate the sum

$$\frac{1}{\sqrt{1} + \sqrt{2}} + \frac{1}{\sqrt{2} + \sqrt{3}} + \frac{1}{\sqrt{3} + \sqrt{4}} + \dots + \frac{1}{\sqrt{15} + \sqrt{16}}.$$

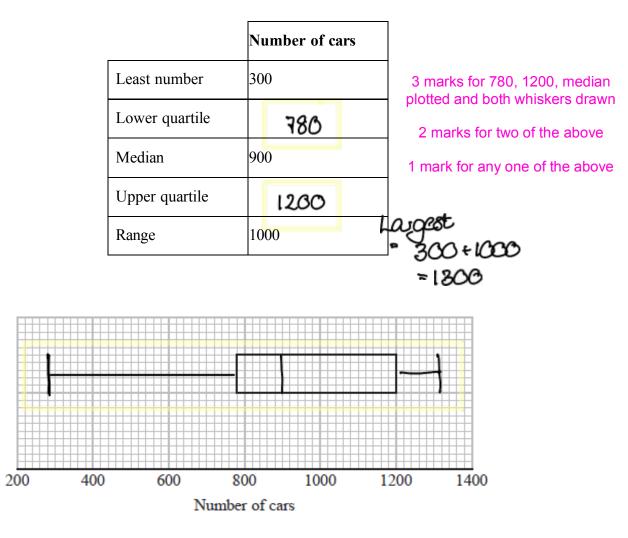
(You might want to use a calculator to get an estimate of the answer, but in order to get the exact answer you will have to do it by hand!)

Can you find a similar sum that evaluates to 5?

Can you find a similar sum that evaluates to a number that is not an integer?

Paper 1 Solutions Answer all questions. Write your answers in the spaces provided. You must write down all the stages in your working.

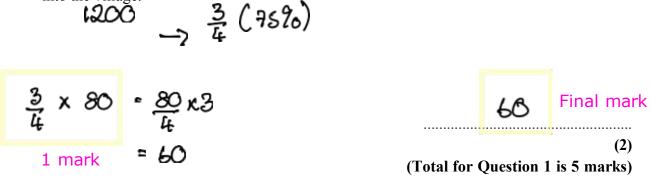
Alice recorded the number of cars going into a village on each of 80 days.
 The incomplete table and the incomplete box plot give information about her results.



(a) (i) Use the information in the table to complete the box plot.(ii) Use the information in the box plot to complete the table.

(3)

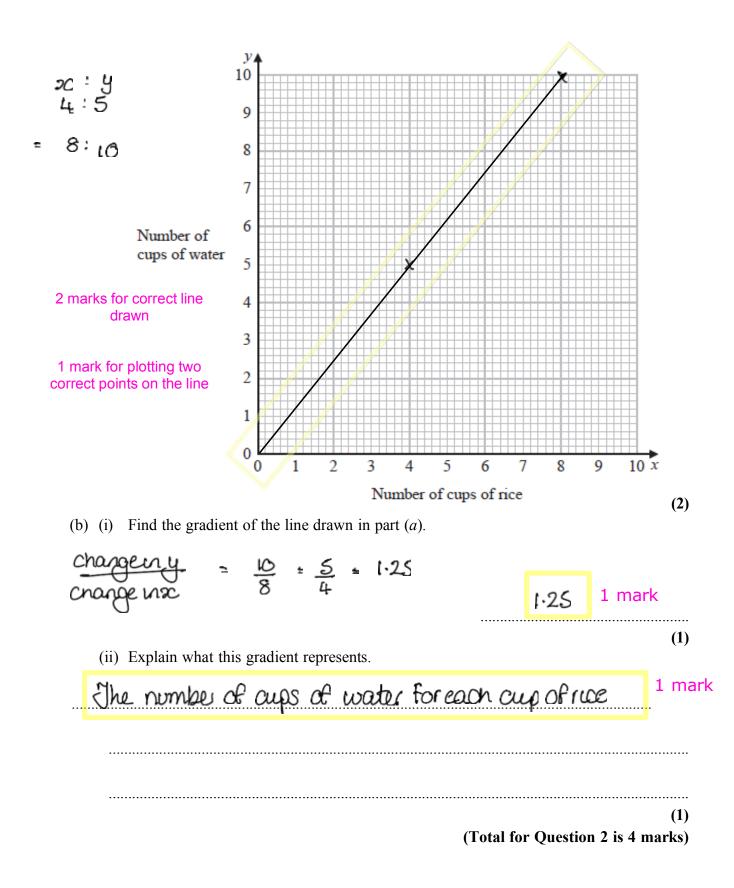
- On some of these 80 days Alice saw fewer than 1200 cars going into the village.
- (b) Work out an estimate for the number of days Alice saw fewer than 1200 cars going into the village.

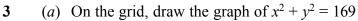


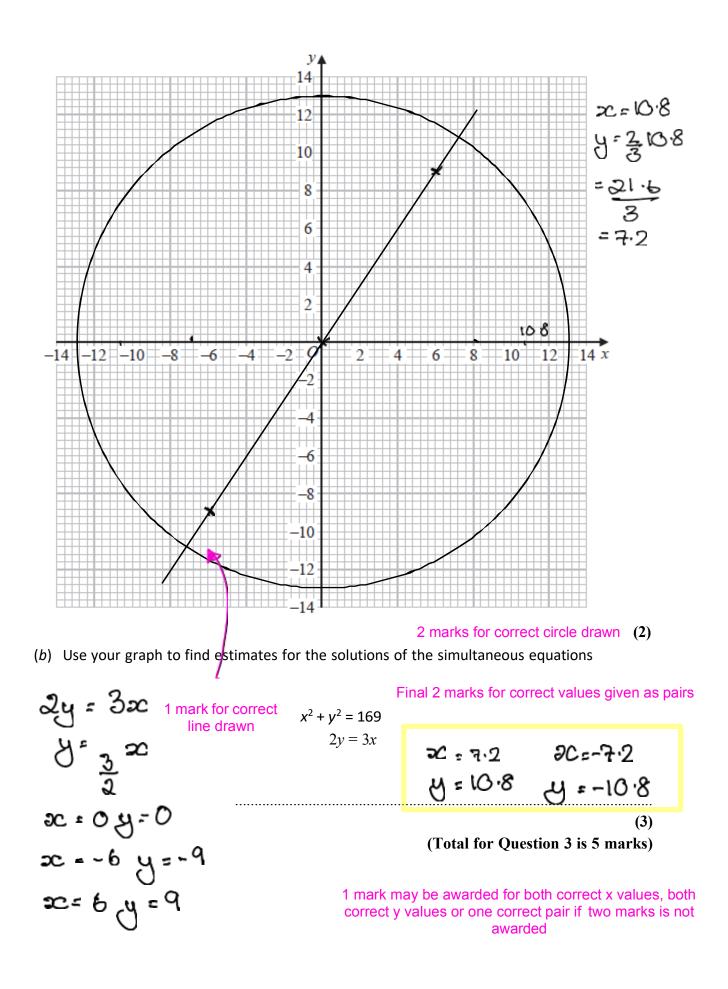
2 To cook rice

the number of cups of rice (x): the number of cups of water (y) = 4:5

(a) Use this information to draw a graph to show the relationship between the number of cups of rice and the number of cups of water needed to cook rice.







7 kg of carrots and 5 kg of tomatoes cost a total of 480p
cost of 1 kg of carrots : cost of 1 kg of tomatoes = 5 : 9
Work out the cost of 1 kg of carrots and the cost of 1 kg of tomatoes.
Corrots : forms
$$\frac{t}{c} = \frac{q}{5}$$

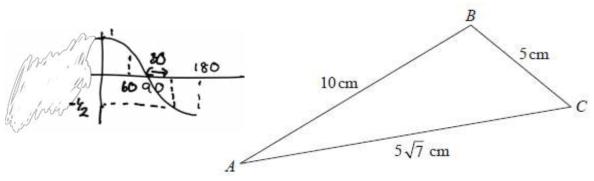
 $5 : q$
 $t = \frac{q}{5}$ 1 mark.
 1 mark .
 $1 \text{ c} + 5t = 480$
 $1 \text{ c} + 5t = 480$
 $1 \text{ c} = \frac{480}{5}$
 $1 \text{ c} = \frac{480}{5}$
 $1 \text{ c} = \frac{480}{5}$
 1 mark .
 $c = \frac{480}{5} = 30$
 $t = 9x$ $30 = \frac{270}{5}$
 $t = 34$
Final mark
 $c = 54$
 $5 \text{ Write } \frac{3\sqrt{3}}{4-\sqrt{3}} - \frac{2}{\sqrt{3}}$ in the form $\frac{a\sqrt{3}+b}{c}$ where a, b and c are integers.

$$\frac{313}{4 - \sqrt{3}} * \frac{4 + \sqrt{3}}{4 + \sqrt{3}} = \frac{12\sqrt{3} + 3\sqrt{3}\sqrt{3}}{16 + 4\sqrt{3} - 4\sqrt{3}} = \frac{12\sqrt{3} + 9}{13}$$

$$= \frac{12\sqrt{3} + 3\sqrt{3}}{16 + 4\sqrt{3} - 4\sqrt{3}} = \frac{2\sqrt{3} + 9}{13}$$

$$= \frac{2\sqrt{3}}{3}$$

$$= \frac{$$

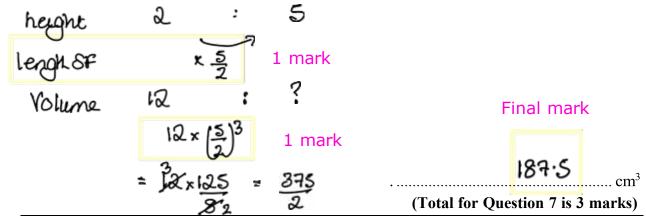


Find the size of angle *ABC*. You must show all your working.

 $c^{2} = a^{2} \cdot b^{2} - 2 ab \cos C$ $(5)^{2} = 10^{2} + 5^{2} - 2 \times 10 \times 5 \times \cos \beta \quad 1 \text{ mark}$ $cos \beta = 5^{2} + 10^{2} - (5\sqrt{7})^{2} \quad 1 \text{ mark}$ $= 125 - 175 \quad 1000$ $= -500 \quad z^{-1} \quad 1 \text{ mark}$ $80 \quad 90 + 30 = 120$ $(see mg gaph at the bop) \qquad 120 \quad \text{Final mark}$

(Total for Question 6 is 4 marks)

7 Solid A and solid B are similar. The ratio of the height of solid A to the height of solid B is 2 : 5 The volume of solid A is 12 cm³ Work out the volume of solid B.
A
B
A
A
A
5

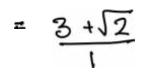


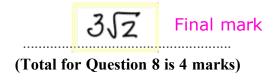
8 The 2nd term of a geometric sequence is $3 + 2 \sqrt{2}$ The 3rd term of the sequence is $13 + 9 \sqrt{2}$ Find the value of the common ratio of the sequence. Give your answer in the form $a + \sqrt{b}$ where *a* and *b* are integers. You must show all your working.

$$\frac{13+9\sqrt{2}}{3+2\sqrt{2}}$$
 1 mark

$$= \frac{13+9\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}} 1 \text{ mark}$$

$$= \frac{39-26\sqrt{2}+27\sqrt{2}}{9-6\sqrt{2}+27\sqrt{2}} - 18\sqrt{2}\sqrt{2} \frac{1}{1} \text{ mark}$$





9 Find the set of possible values of x for which $4x^2 - 25 < 0$ and $12 - 5x - 3x^2 > 0$ You must show all your working.

$$(2x-5)(yx+5)<0 \text{ 1 mark}$$

$$\frac{5}{3} \quad \frac{-5}{3}$$

$$2\cdot5 \quad -25 \quad 1 \text{ mark}$$

$$-3x^2 - 5x + ky>0 \quad -36$$

$$4,9$$

$$-3x^2 - 9x + 4xc + 12>0$$

$$-3x(x+3) + 4(x+3)>0$$

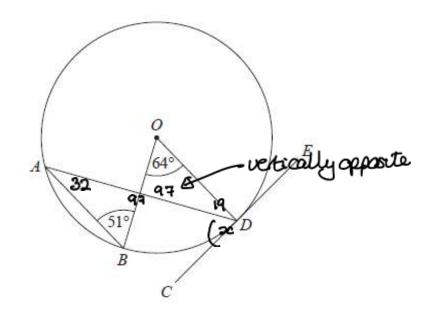
$$(-3x+4)(x+3)>0 \quad 1 \text{ mark}$$

$$\frac{4}{3} \mid \frac{1}{3} \quad -3 \quad 1 \text{ mark}$$

$$x < \frac{4}{3} \quad x > -3 \quad 80 \quad -2 \le \frac{02}{5}$$
Final mark
$$-2\cdot5(x < \frac{4}{3})$$

(Total for Question 9 is 5 marks)

10 *A*, *B* and *D* are points on a circle with centre *O*. *CDE* is the tangent to the circle at *D*.



Work out the size of angle *ADC*. Write down any circle theorems you use.

het ADC =
$$\infty$$

1 mark $BAD = 32$ angle at the centress have the angle at the curve of (BAD)
 $1 mark BAD = 32$ angle at the centres have the angle of (BAD)
 $1 mark BAD = 32$ angle at the centres have the angle = 180
 $180 - (51 + 32) = 180 - 83 = 97$
 $1 mark BAD = 180 - (97 + 64)^{\pm 161}$
 $\infty = 90 - 19 = 71$ angle between tangent and radius = 90^{-19}
Final mark awarded for correct reasons

1 mark

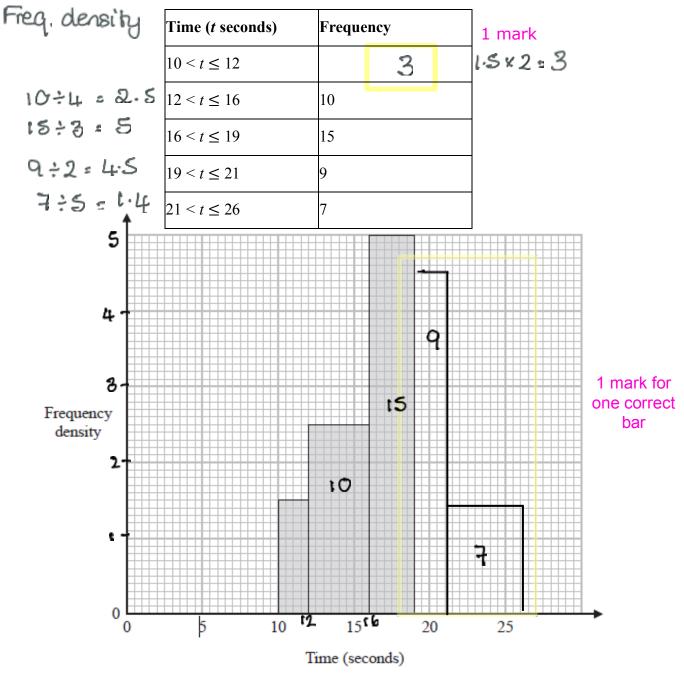
71

Paper 2 Solutions Answer all questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The incomplete table and the incomplete histogram give information about the times taken by some students to run a race.



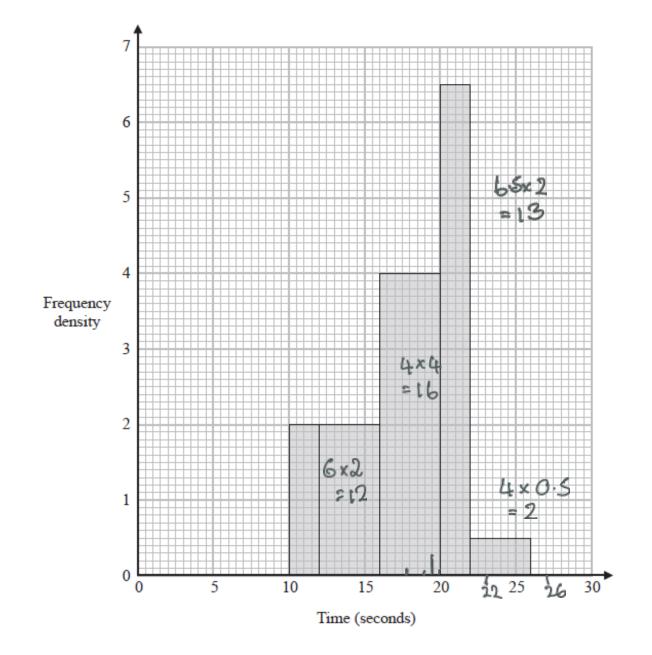
None of these students had a time for the race such $\int 10$ or t > 26 that t

(a) Use the histogram to complete the table.

(1)

(b) Use the table to complete the histogram.

The histogram below gives information about the times taken by 43 students to run a different race.



(c) Work out an estimate for the median of the times taken by these 43 students to run the 1 mark Total $^{race.}$ = 12 + 16 + 13 + 2 = 43 Meduan = 43 + 1 = 22 nd Final mark

$$Median = \frac{43+1}{2} = 22nd \qquad Final mark$$

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seconds	18.5	
(3)		

1 mark

(Total for Question 1 is 6 marks)

2 A biased dice is thrown 60 times.

The table shows information about the number that the dice lands on each time.

Number on dice	1	2	3	4	5	6
Frequency	12	7	8	9	9	15

Gethin throws the dice twice.

(a) Work out an estimate for the probability that the dice will land on 6 both times.

$$1 \text{ mark } P(6) = \frac{15}{60} = \frac{1}{4}$$

$$P(6,6) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$$

$$I \text{ mark } = \frac{1}{16} \qquad \frac{1}{16} \begin{pmatrix} 225 \\ 3600 \end{pmatrix}$$

$$\frac{1}{16} \begin{pmatrix} 225 \\ 3600 \end{pmatrix}$$

$$(3)$$

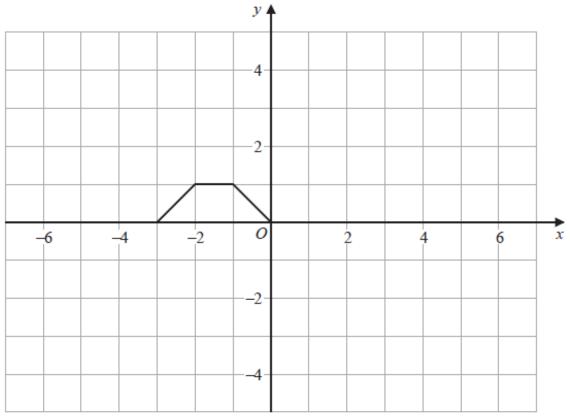
Sally is going to throw the same dice *n* times and record the number it lands on each time. She will use her results to work out a more reliable estimate for the probability in part (*a*).

(b) What can you say about the value of *n*?

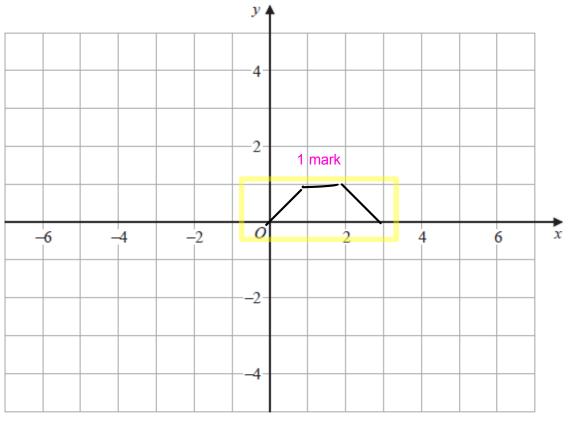
..... greater the value of n, the more reliaitle her 1 its will be so n should be greater than 60 1 mark

(Total for Question 2 is 4 marks)

3 Here is the graph of y = f(x)

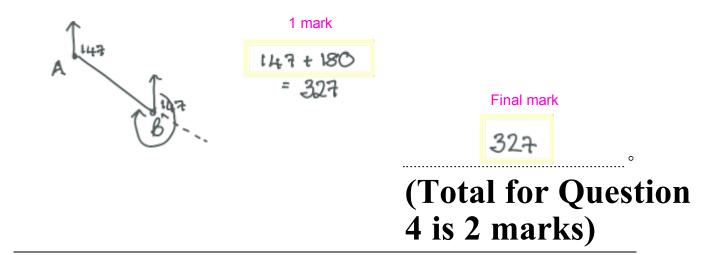


On the grid below, draw the graph of y = f(-x)



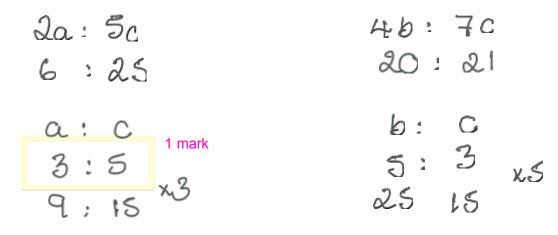
(Total for Question 3 is 1 mark)

*4 The bearing of port *B* from port *A* is 147°Work out the bearing of port *A* from port *B*.



5 2a: 5c = 6: 254b: 7c = 20: 21

Show that a + b : b + c = 17 : 20



a: b: c
1 mark
$$9: ds: 15$$

Final mark
 $a+b = 34$ $b+c = 40$
 $34: 40$
 $= 17: 20$ as required.

6 Write $\begin{array}{c} 14 \\ 3x-21 \end{array} + \begin{bmatrix} 2x^2 - 6x - 56 \end{bmatrix}$ ax + b $2x+3 \\ 2x+3 \\ \end{bmatrix}$ in the form $\overline{cx+d}$ where a, b, c and d are integers.

$$2x^2 - 6x - 56 = (x + 4)(2x - 14)$$
 1 mark

$$\frac{14}{3x-21} + (x+4) \times \frac{2x+3}{2x^2-6x-56}$$

1 mark

$$= \frac{14}{32c-21} + \frac{(2c+4)(22c+1)}{(2c+4)(22c-1)}$$

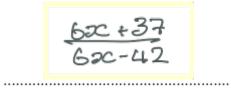
$$= \frac{14(2\pi-14) + (2\pi+3)(3\pi-21)}{(3\pi-21)(2\pi-14)}$$
 1 mark

$$= 28\pi - 196 + 6\pi^2 - 62\pi + 9\pi - 63$$

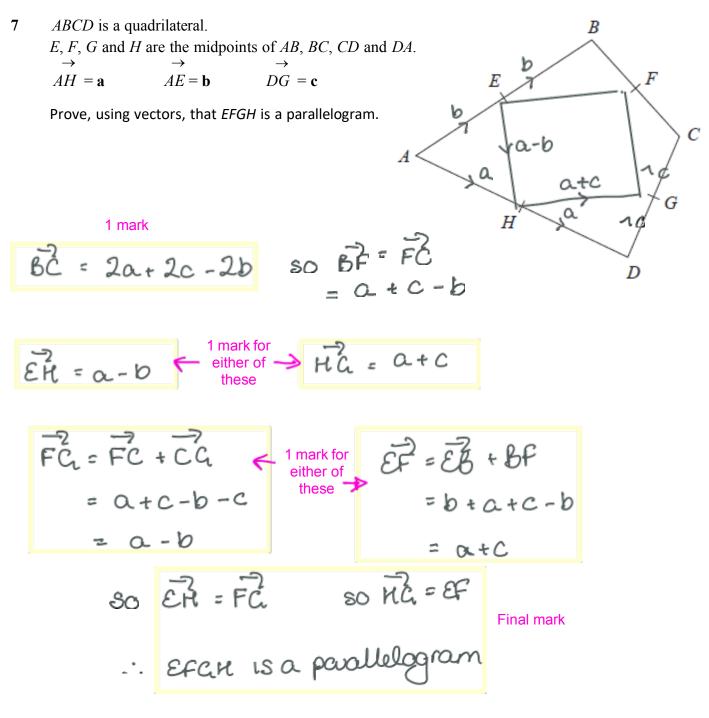
$$(3\pi - 21)2(\pi - 7)$$

$$= \frac{62c + 37}{62c - 42}$$

Final mark



(Total for Question 6 is 4 marks)

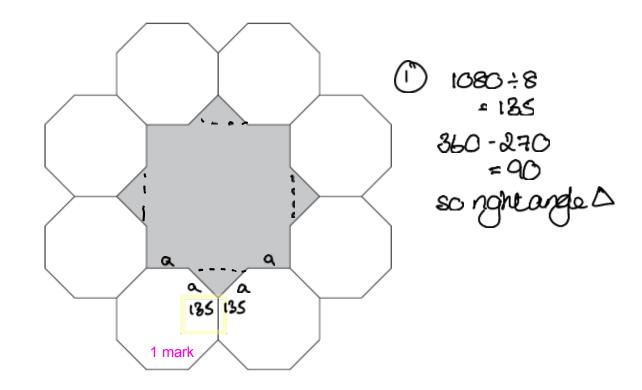


(Total for Question 7 is 4 marks)

8 Show that the equation $x^3 + 2x - 6 = 0$ has a solution between x = 1 and x = 2

$$2C = 1 \quad \begin{array}{c} 13 + 2 \times 1 - 6 = 1 + 2 - 6 = -3 \\ 1 \text{ mark for either of these} \\ 2C = 2 \quad 2^3 + 2 \times 2 - 6 = 8 + 4 - 6 = 6 \\ \hline \\ \text{Final mark} \\ \text{Since there is a change in sign a solution} \\ \text{must be between } x = 1 \text{ ord } x = 2 \\ \end{array}$$

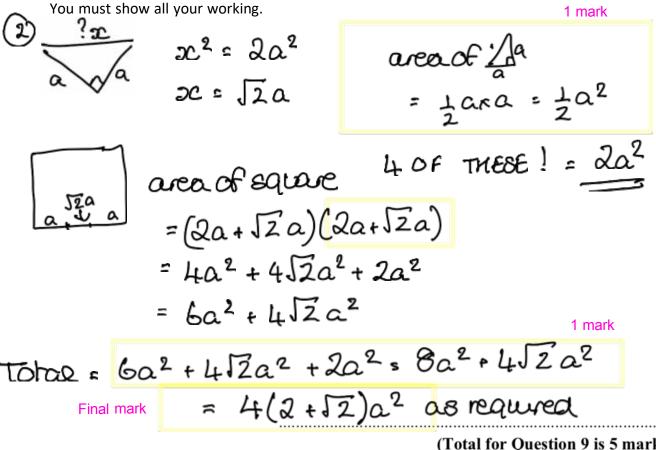
9 The diagram shows 8 identical regular octagons joined to enclose a shaded shape.



Each octagon has sides of length a.

Find, in terms of a, an expression for the area of the shaded shape.

Give your answer in the form $p(2 + \sqrt{2})a^2$ where p is an integer.

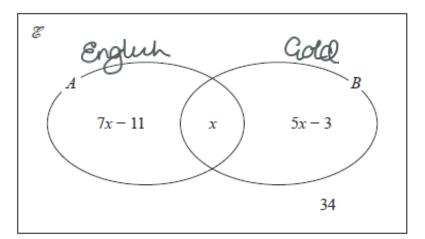


(Total for Question 9 is 5 marks)

10 Vicky has a collection of medals.

The Venn diagram gives information about the number of medals in her collection where

- &= {all medals}
- A = {English medals}
- B = {gold medals}



Vicky is going to take at random a medal from her collection.

Given that the medal is gold, the probability that the medal is English is $\frac{2}{11}$

Work out the number of medals in Vicky's collection.

<u>2</u> L (

2

1 mark

$$112c = 2(6x-3)$$

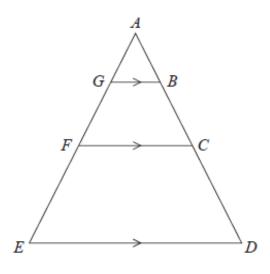
$$112c = 122c - 6$$

$$2c = 6$$

$$30 \text{ medlols} = 13 \times 6 + 20$$

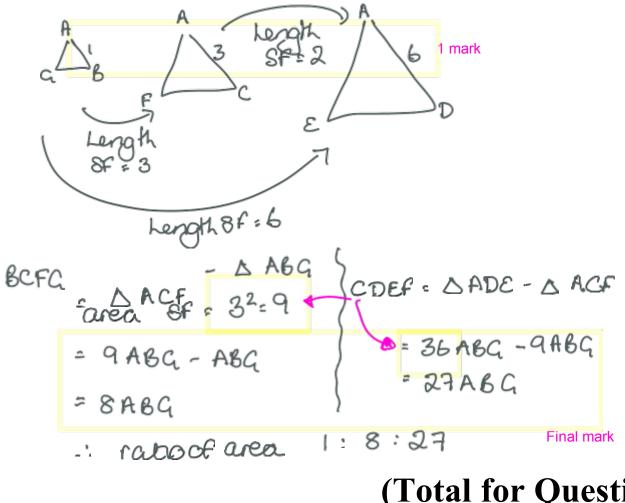
$$98$$

11 Here are three similar triangles, *ABG*, *ACF* and *ADE*.



ABCD and AGFE are straight lines. AB: BC: CD = 1:2:3Show that

area of ABG : area of BCFG : area of CDEF = 1 : 8 : 27

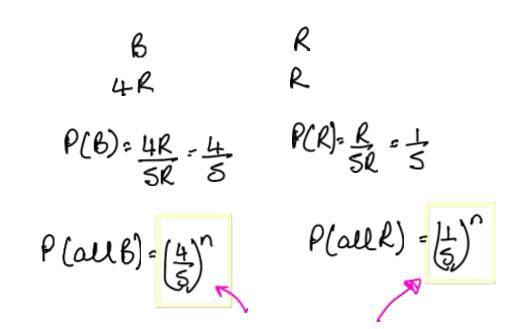


(Total for Question 11 is 3 marks) 12 There are only blue pens and red pens in a box.

The number of blue pens is four times the number of red pens. Rita takes at random one pen from the box.

She records the colour of the pen and then replaces it in the box. Rita does this *n* times, where $n \ge 2$

Write down an expression, in terms of n, for the probability that Rita gets a blue pen at least once and a red pen at least once.



1 mark for either of these

$$(-(\frac{4}{5})^{2}-(\frac{1}{5})^{2})$$

Paper 3 Solutions Answer all questions.

Write your answers in the spaces provided. You must write down all the stages in your working.

(a) Factorise $a^2 - b^2$ 1

(a-b)(a+b)

(b) Show that $2^{40} - 1$ is the product of two consecutive odd numbers.

$$a^2 - b^2$$
 so $(2^{20})^2 - 1^2$

$$= (2^{20} - 1)(2^{20} + 1) 1 \text{ mark}$$

$$\mathcal{J}^{20} = even \quad \mathcal{Z}^{20} - 1 = odd$$

so $\mathcal{J}^{20} + 1$ is next consecutive odd number.

(2)

1 mark

(a-b)(a+b)

(1)

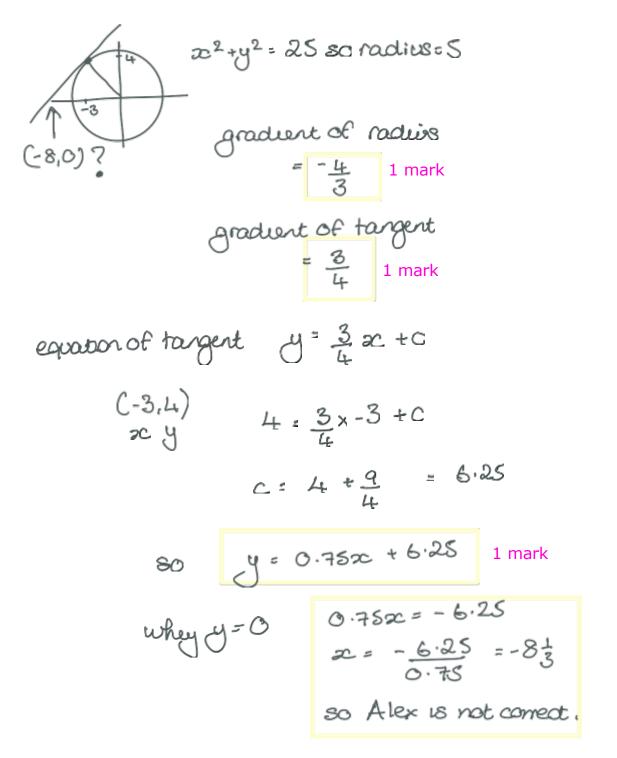
2 A circle has equation $x^2 + y^2 = 25$

The point *P* with coordinates (-3, 4) lies on the circle.

Alex says that the tangent to the circle at P crosses the x-axis at the point (-8,

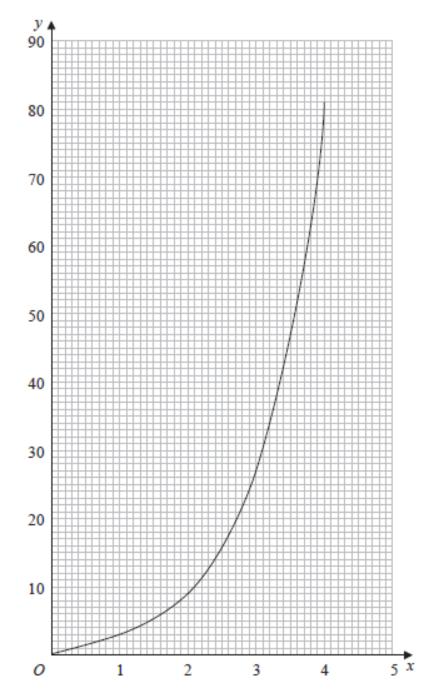
0) Is Alex correct?

You must show how you get your answer.



3 Sana needs to draw the graph of $y = 3^x$ for $0 \le x \le 4$

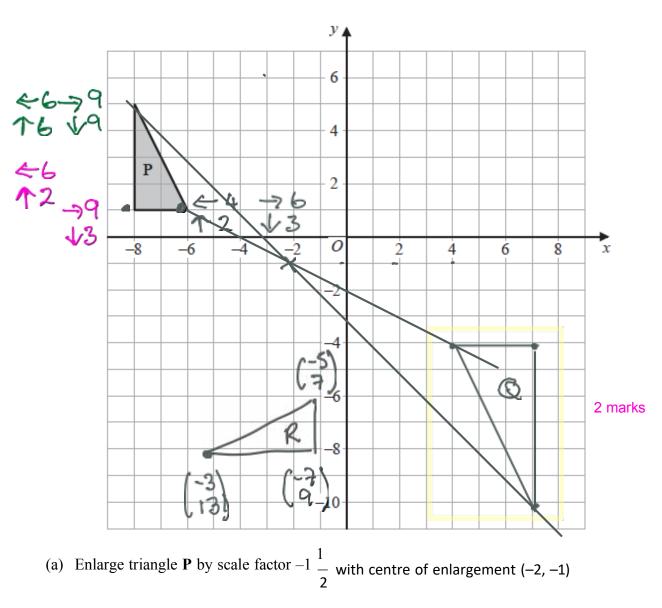
She draws the graph shown on the grid.



Write down one thing Sana has done wrong.

1 mark When oc = O y = 3° = 1 and not zero. so the line should go through (0,1) not (0,0)





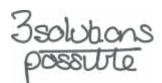
Label your image **Q**.

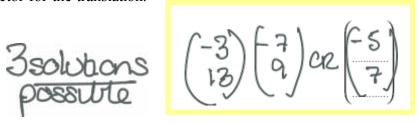
(2)

1 mark for

any one of these

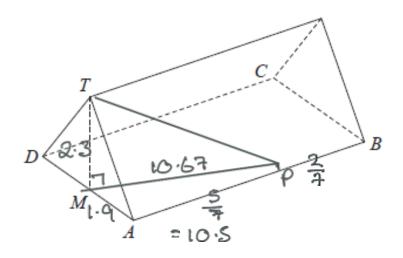
Triangle P is transformed by a combined transformation of a rotation of 90° anticlockwise about the origin followed by a translation to give triangle R. Exactly one vertex of triangle **P** is invariant under the combined transformation. (b) Find one possible column vector for the translation.





(1)

5 The diagram shows a triangular prism with a horizontal rectangular base *ABCD*.



M is the midpoint of *AD*.

The vertex T of the prism is vertically above M.

AB = 14.7 cm BC = 3.8 cm MT = 2.3 cm

P is the point on AB such that

$$AP:PB=5:2$$

Calculate the size of the angle between *TP* and the base *ABCD* of the prism. Give your answer correct to 1 decimal place.

$$AP = \frac{5}{7} \times 14.7 \times 10.5 \quad 1 \text{ mark}$$

$$MP^{2} = \int 10.52 \pm 1.92 \quad 1 \text{ mark}$$

$$= \int 113.86$$

$$= 10.67$$

$$\tan TPM = \frac{2.3}{10.67} \quad 1 \text{ mark}$$

$$TPM = \tan^{-1} \frac{2.3}{10.67} \quad Final \text{ mark}$$

$$= 12.1638..$$

6 An expression for the *n*th term of the sequence of triangular numbers is $\frac{n(n+1)}{2}$ Prove that the sum of any two consecutive triangular numbers is a square number.

$$= \frac{n(n+1)}{2} + (n+1)(n+1+1) = (n+1)^{2}$$

$$= \frac{n^{2}+n}{2} + (n+1)(n+2) = (n+1)^{2}$$

$$= \frac{n^{2}+n+n^{2}+3n+2}{2} = 2n^{2}+4n+2$$

$$= n^{2}+2n+1 = (n+1)(n+1) = (n+1)^{2}$$
Final mark
which is a square number.

The floor plan of a house is drawn using a scale of 1 : 50
 On the plan, a room in the house has a floor area of 48 cm²
 Work out the real area of the floor of this room.
 Give your answer in m²

$$length \ \delta F = 50 \ 1 \ mark$$
so area $\delta F = 50^{2} = 2500$

$$\lim_{m \to \infty} 4.8 \times 2500 = 120\ 000\ cm^{2}$$

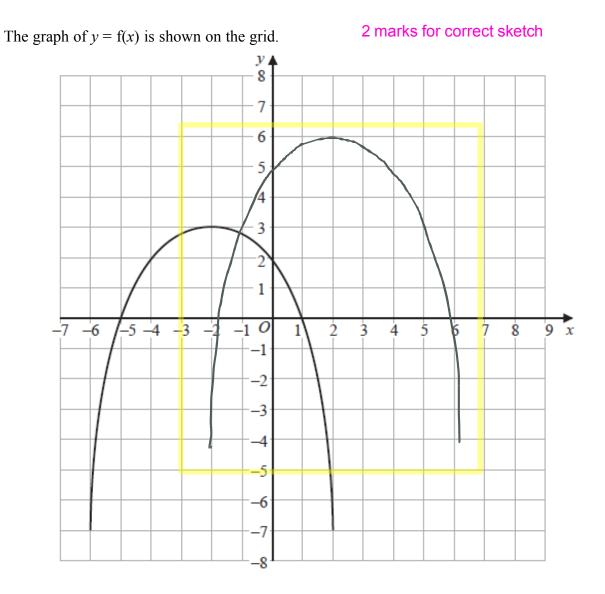
$$\lim_{m \to \infty} 1m^{2} = 10,000\ cm^{2}$$

$$\lim_{m \to \infty} 120000\ cm^{2} \div 10000\ cm^{2}$$

$$= 12$$

$$\lim_{m \to \infty} 12\ 12\ \text{Final mark}$$

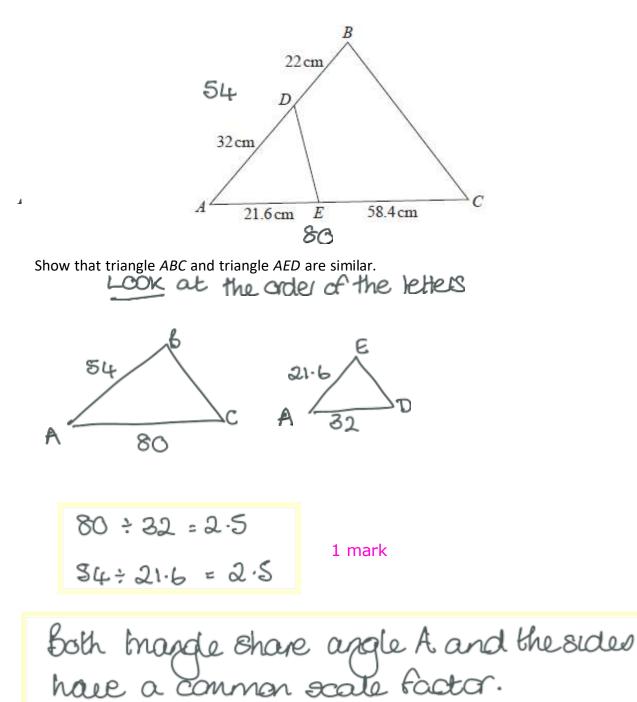
$$m^{2}$$



On the grid, sketch the graph of y = f(-x) + 3

8

The diagram shows triangle ABC and triangle AED. 9



10 There is a total of *y* counters in a box.

There are *x* pink counters and 5 blue counters in the box.

The rest of the counters are green.

x: y = 1:3

Freda takes at random two counters from the box.

Find, in terms of *x*, an expression for the probability that Freda takes two counters of the same colour.

Give your answer as a fraction in the form
$$\frac{ax^2+bx+c}{dx^2+ex}$$
 where a, b, c, d and e are integers.
1 mark

 $\frac{2c}{5} = \frac{1}{3}$

11 Ebony makes some bracelets to sell.

The materials to make all the bracelets cost £190, correct to the nearest £5

Ebony sells all the bracelets for a total of £875, correct to the nearest £5

The total time taken to make and sell all these bracelets was 72 hours, correct to the nearest hour.

Ebony uses this method to calculate her hourly rate of pay

Hourly rate of pay = $\frac{\text{total selling price} - \text{total cost of materials}}{\text{total time taken}}$

The minimum hourly rate of pay for someone of Ebony's age is £8.20

By considering bounds, determine if Ebony's hourly rate of pay was definitely more than

£8.20

You must show all your working.

materials 190 necrest
$$#5$$

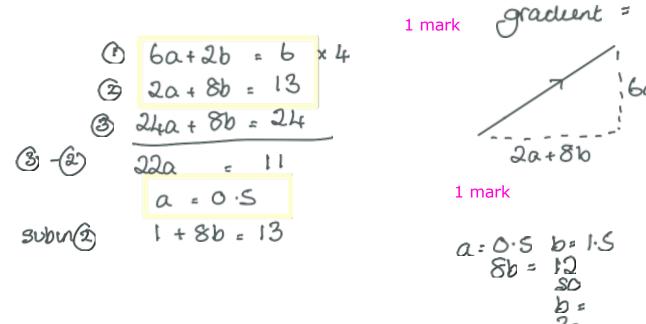
sells 875 nearest $\pounds 5$
 1102.50
 2187.50
 7877.50
 872.50
 872.50
 1772.5
true 72 hows nearest har 772.5
 271.5

1 mark for any one of these bounds

Looking for LB of hourly rate of pay,
=
$$\begin{bmatrix} 872.50 - 192.50 \\ 72.50 \end{bmatrix}$$
 1 mark
= 9.3751.. So £9.38
Yes it is definitely more than
£.8.20

Given that the vector $a \begin{pmatrix} 2 \\ 6 \end{pmatrix} + b \begin{pmatrix} 8 \\ 2 \end{pmatrix}$ is parallel to the vector $\begin{pmatrix} 13 \\ 6 \end{pmatrix}$ /- ıŝ 12

find an expression for *b* in terms of *a*.



16

6

Final mark