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Exercise 1:
Simplifying indices

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the *n*th root of *a*
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \dfrac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Example 1 Simplify $\dfrac{6x^5}{2x^2}$

$\dfrac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\dfrac{a^m}{a^n} = a^{m-n}$ to give $\dfrac{x^5}{x^2} = x^{5-2} = x^3$

Example 2 Simplify $\dfrac{x^3 \times x^5}{x^4}$

$\dfrac{x^3 \times x^5}{x^4} = \dfrac{x^{3+5}}{x^4} = \dfrac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\dfrac{a^m}{a^n} = a^{m-n}$

Example 3 Write $\dfrac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged

Example 4 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\frac{1}{a^m} = a^{-m}$

Practice questions

1 Simplify.

a $\frac{3x^2 \times x^3}{2x^2}$ **b** $\frac{10x^5}{2x^2 \times x}$

c $\frac{3x \times 2x^3}{2x^3}$ **d** $\frac{7x^3y^2}{14x^5y}$

e $\frac{y^2}{y^{\frac{1}{2}} \times y}$ **f** $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g $\frac{(2x^2)^3}{4x^0}$ **h** $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

> **Watch out!**
>
> Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

2 Write the following as a single power of x.

a $\frac{1}{x}$ **b** $\frac{1}{x^7}$ **c** $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$ **e** $\frac{1}{\sqrt[3]{x}}$ **f** $\frac{1}{\sqrt[3]{x^2}}$

3 Write the following without negative or fractional powers.

a x^{-3} **b** x^0 **c** $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{2}}$ **f** $x^{-\frac{3}{4}}$

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Answers

1 **a** $\frac{3x^3}{2}$ **b** $5x^2$

c $3x$ **d** $\frac{y}{2x^2}$

e $y^{\frac{1}{2}}$ **f** c^{-3}

g $2x^6$ **h** x

2 **a** x^{-1} **b** x^{-7} **c** $x^{\frac{1}{4}}$

d $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{3}}$ **f** $x^{-\frac{2}{3}}$

3 **a** $\frac{1}{x^3}$ **b** 1 **c** $\sqrt[5]{x}$

d $\sqrt[5]{x^2}$ **e** $\frac{1}{\sqrt{x}}$ **f** $\frac{1}{\sqrt[4]{x^3}}$

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Exercise 2: Simplifying fractions

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the nth root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	**1** Use the rule $a^{-m} = \frac{1}{a^m}$ **2** Use $4^2 = 16$

Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$

Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$

Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged

Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\frac{1}{a^m} = a^{-m}$

Example 9 Simplify $\dfrac{x^5}{x^2}$

$\dfrac{x^5}{x^2} = x^3$	use the rule $\dfrac{a^m}{a^n} = a^{m-n}$ to give $\dfrac{x^5}{x^2} = x^{5-2} = x^3$

Example 10 Simplify $6x^6 \times 3x^4$

$6x^6 \times 3x^4 = 18x^2$	$6 \times 3 = 18$ and then use the rule $a^m \times a^n = a^{m+n}$ to give $x^6 \times x^4 = x^{6+4} = x^{10}$

Example 11 Simplify $(x^4)^2 \times 3x^5$

$(x^4)^2 \times 3x^5 = 3x^{13}$	$3 \times 1 = 3$ and then use the rule $(a^m)^n = a^{mn}$ following by to give $a^m \times a^n = a^{m+n}$ $(x^4)^2 \times x^5 = x^{4\times 2} \times x^5$ $= x^8 \times x^5$ $= x^{8+5}$ $= x^{13}$

Practice questions

1 Write as sums of powers of x.

a $\dfrac{x^5+1}{x^2}$ **b** $x^2\left(x - \dfrac{1}{x}\right)$ **c** $x^{-4}\left(x^2 + \dfrac{1}{x^3}\right)$

d $\dfrac{6x^5+3x^4}{3x^2}$ **e** $\dfrac{5x^5+20x^4}{10x^2}$ **f** $\dfrac{7x^5-5x^4}{2x^6}$

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Answers

1 **a** $x^3 + x^{-2}$ **b** $x^3 - x$ **c** $x^{-2} + x^{-7}$

d $2x^3 + x^2$ e $0.5x^3 + 2x^2$ f $3.5x^{-1} - 2.5x^{-2}$

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Exercise 3: Expanding brackets and surds

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5},$ etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Example 1 Simplify $(\sqrt{7}+\sqrt{2})(\sqrt{7}-\sqrt{2})$

$(\sqrt{7}+\sqrt{2})(\sqrt{7}-\sqrt{2})$ $= \sqrt{49}-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}-\sqrt{4}$ $= 7-2$ $= 5$	**1** Expand the brackets. A common mistake here is to write $(\sqrt{7})^2 = 49$ **2** Collect like terms: $-\sqrt{7}\sqrt{2}+\sqrt{2}\sqrt{7}$ $= -\sqrt{7}\sqrt{2}+\sqrt{7}\sqrt{2} = 0$

Practice questions

1 Expand and simplify.

a $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$

b $(3+\sqrt{3})(5-\sqrt{12})$

c $(4-\sqrt{5})(\sqrt{45}+2)$

d $(5+\sqrt{2})(6-\sqrt{8})$

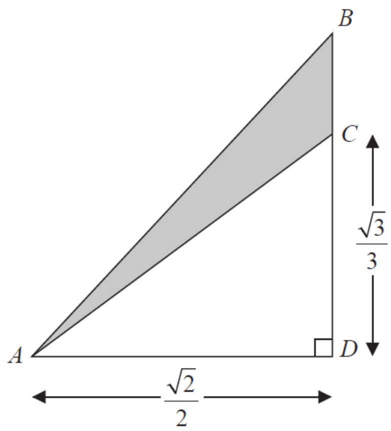
2 Expand and simplify $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

3 Work out the value of $\left(\sqrt{2}+\sqrt{8}\right)^2$

4 Expand $(1+\sqrt{2})(3-\sqrt{2})$

Give your answer in the form $a + b\sqrt{2}$ where a and b are integers.

5 *ABD* is a right angled triangle.



All measurements are given in centimetres.

C is the point on *BD* such that $CD = \dfrac{\sqrt{3}}{3}$

$AD = BD = \dfrac{\sqrt{2}}{2}$

Work out the exact area, in cm^2, of the shaded region.

6 The diagram shows a triangle *DEF* inside a rectangle *ABCD*.

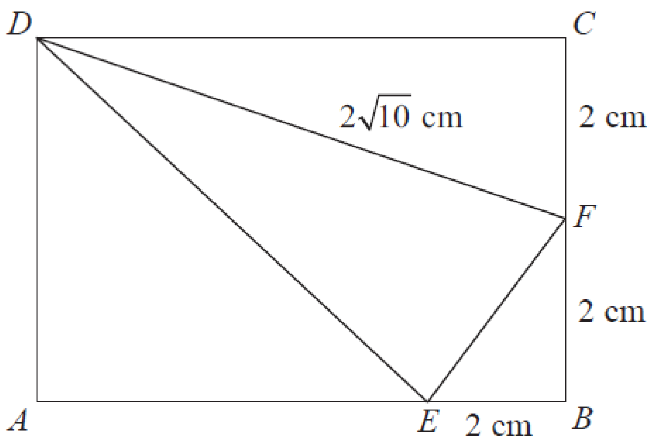


Diagram **NOT** accurately drawn

Show that the area of triangle *DEF* is 8 cm^2.
You must show all your working.

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7 The diagram shows a triangle *DEF* inside a rectangle *ABCD*.

$$a = \sqrt{8} + 2$$
$$b = \sqrt{8} - 2$$
$$T = a^2 - b^2$$

Work out the value of *T*.

Give your answer in the form $c\sqrt{2}$ where *c* is an integer.

....

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Answers

1 **a** -1 **b** $9-\sqrt{3}$

c $10\sqrt{5}-7$ **d** $26-4\sqrt{2}$

2 $x-y$

3 18

4 $1+2\sqrt{2}$

5 $\frac{1}{4}-\frac{\sqrt{6}}{12}$

Method:

$\frac{1}{2}\times\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2}$ or $\frac{1}{2}\times\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{3}$

$\frac{1}{2}\times\frac{\sqrt{2}}{2}\times\frac{\sqrt{2}}{2}-\frac{1}{2}\times\frac{\sqrt{2}}{2}\times\frac{\sqrt{3}}{3}$

$\frac{1}{4}-\frac{\sqrt{6}}{12}$ oe

OR

(BC =) $\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{3}$

$\frac{1}{2}\times\left\{\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{3}\right\}\times\frac{\sqrt{2}}{2}$

$\frac{1}{4}-\frac{\sqrt{6}}{12}$ oe

6 8

Method:

$(2\sqrt{10})^2-2^2$ (= 36)

(CD =) 6

$`6' \times 4-\frac{1}{2}\times `6' \times 2-\frac{1}{2}\times 2\times 2-\frac{1}{2}\times(`6'-2)\times 4$

7 $16\sqrt{2}$

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Exercise 4:
Factorising quadratics

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Example 1 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and −2)
So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$	**2** Rewrite the b term ($3x$) using these two factors
$= x(x + 5) - 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$= (x + 5)(x - 2)$	**4** $(x + 5)$ is a factor of both terms

Example 2 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (−15 and 4)
So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$	**2** Rewrite the b term ($-11x$) using these two factors
$= 3x(2x - 5) + 2(2x - 5)$	**3** Factorise the first two terms and the last two terms
$= (2x - 5)(3x + 2)$	**4** $(2x - 5)$ is a factor of both terms

Example 3 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

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Practice questions

1 Factorise

a	$x^2 + 7x + 12$	**b**	$x^2 + 5x - 14$
c	$x^2 - 11x + 30$	**d**	$x^2 - 5x - 24$
e	$x^2 - 7x - 18$	**f**	$x^2 + x - 20$
g	$x^2 - 3x - 40$	**h**	$x^2 + 3x - 28$

2 Factorise fully

a	$y^2 - 100$	**b**	$36x^2 - 49y^2$
c	$4x^2 - 81y^2$	**d**	$18a^2 - 200b^2c^2$

3 Factorise fully

a	$2x^2 + x - 3$	**b**	$6x^2 + 17x + 5$
c	$2x^2 + 7x + 3$	**d**	$9x^2 - 15x + 4$
e	$10x^2 + 21x + 9$	**f**	$12x^2 - 38x + 20$

Answers

1	**a**	$(x + 3)(x + 4)$	**b**	$(x + 7)(x - 2)$
	c	$(x - 5)(x - 6)$	**d**	$(x - 8)(x + 3)$
	e	$(x - 9)(x + 2)$	**f**	$(x + 5)(x - 4)$
	g	$(x - 8)(x + 5)$	**h**	$(x + 7)(x - 4)$
2	**a**	$(y - 10)(y + 10)$	**b**	$(6x - 7y)(6x + 7y)$
	c	$(2x - 9y)(2x + 9y)$	**d**	$2(3a - 10bc)(3a + 10bc)$
3	**a**	$(x - 1)(2x + 3)$	**b**	$(3x + 1)(2x + 5)$
	c	$(2x + 1)(x + 3)$	**d**	$(3x - 1)(3x - 4)$
	e	$(5x + 3)(2x + 3)$	**f**	$2(3x - 2)(2x - 5)$

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Exercise 5:
Solving quadratic equations

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$.
$5x(x - 3) = 0$	**2** Factorise the quadratic equation. $5x$ is a common factor.
So $5x = 0$ or $(x - 3) = 0$	**3** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = 0$ or $x = 3$	**4** Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3)
$x^2 + 4x + 3x + 12 = 0$	**2** Rewrite the b term ($7x$) using these two factors.
$x(x + 4) + 3(x + 4) = 0$	**3** Factorise the first two terms and the last two terms.
$(x + 4)(x + 3) = 0$	**4** $(x + 4)$ is a factor of both terms.
So $(x + 4) = 0$ or $(x + 3) = 0$	**5** When two values multiply to make zero, at least one of the values must be zero.
Therefore $x = -4$ or $x = -3$	**6** Solve these two equations.

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Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\frac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3) **2** Rewrite the b term ($-5x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Practice questions

1 Solve

a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$

c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$

e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$

i $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$

k $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$

c $x^2 + 5x = 24$ **d** $x^2 - 42 = x$

e $x(x + 2) = 2x + 25$ **f** $x^2 - 30 = 3x - 2$

g $x(3x + 1) = x^2 + 15$ **h** $3x(x - 1) = 2(x + 1)$

Hint

Get all terms onto one side of the equation.

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Answers

1 **a** $x = 0$ or $x = -\frac{2}{3}$

b $x = 0$ or $x = \frac{3}{4}$

c $x = -5$ or $x = -2$

d $x = 2$ or $x = 3$

e $x = -1$ or $x = 4$

f $x = -5$ or $x = 2$

g $x = 4$ or $x = 6$

h $x = -6$ or $x = 6$

i $x = -7$ or $x = 4$

j $x = 3$

k $x = -\frac{1}{2}$ or $x = 4$

l $x = -\frac{2}{3}$ or $x = 5$

2 **a** $x = -2$ or $x = 5$

b $x = -1$ or $x = 3$

c $x = -8$ or $x = 3$

d $x = -6$ or $x = 7$

e $x = -5$ or $x = 5$

f $x = -4$ or $x = 7$

g $x = -3$ or $x = 2\frac{1}{2}$

h $x = -\frac{1}{3}$ or $x = 2$

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Exercise 6:
Simplifying algebraic fractions

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Example 1 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$ So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$ $= x(x + 5) - 2(x + 5)$ $= (x + 5)(x - 2)$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and −2) **2** Rewrite the b term ($3x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(x + 5)$ is a factor of both terms

Example 2 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$ So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (−15 and 4) **2** Rewrite the b term ($-11x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(2x - 5)$ is a factor of both terms

Example 3 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 4 Simplify $\dfrac{x^2-4x-21}{2x^2+9x+9}$

$\dfrac{x^2-4x-21}{2x^2+9x+9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3)
So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$	**3** Rewrite the b term ($-4x$) using these two factors
$= x(x - 7) + 3(x - 7)$	**4** Factorise the first two terms and the last two terms
$= (x - 7)(x + 3)$	**5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3)
So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$	**7** Rewrite the b term ($9x$) using these two factors
$= 2x(x + 3) + 3(x + 3)$	**8** Factorise the first two terms and the last two terms
$= (x + 3)(2x + 3)$	**9** $(x + 3)$ is a factor of both terms
So $\dfrac{x^2-4x-21}{2x^2+9x+9} = \dfrac{(x-7)(x+3)}{(x+3)(2x+3)}$ $= \dfrac{x-7}{2x+3}$	**10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

Practice questions

1 Simplify the algebraic fractions.

a $\frac{2x^2+4x}{x^2-x}$

b $\frac{x^2+3x}{x^2+2x-3}$

c $\frac{x^2-2x-8}{x^2-4x}$

d $\frac{x^2-5x}{x^2-25}$

e $\frac{x^2-x-12}{x^2-4x}$

f $\frac{2x^2+14x}{2x^2+4x-70}$

2 Simplify

a $\frac{9x^2-16}{3x^2+17x-28}$

b $\frac{2x^2-7x-15}{3x^2-17x+10}$

c $\frac{4-25x^2}{10x^2-11x-6}$

d $\frac{6x^2-x-1}{2x^2+7x-4}$

Extend

3 Simplify $\sqrt{x^2+10x+25}$

4 Simplify $\frac{(x+2)^2+3(x+2)^2}{x^2-4}$

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Answers

1 **a** $\dfrac{2(x+2)}{x-1}$

b $\dfrac{x}{x-1}$

c $\dfrac{x+2}{x}$

d $\dfrac{x}{x+5}$

e $\dfrac{x+3}{x}$

f $\dfrac{x}{x-5}$

2 **a** $\dfrac{3x+4}{x+7}$

b $\dfrac{2x+3}{3x-2}$

c $\dfrac{2-5x}{2x-3}$

d $\dfrac{3x+1}{x+4}$

3 $(x+5)$

4 $\dfrac{4(x+2)}{x-2}$

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Exercise 7: Complete the square

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r$

Examples

Example 1 Complete the square for the expression $x^2 + 6x$

$x^2 + 6x$ $= \left(x + \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2$ $= (x + 3)^2 - 9$	**1** Write $x^2 + bx + c$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$ **2** Simplify.

Example 2 Complete the square for the expression $2x^2 - 7x$

$2x^2 - 7x$ $= 2\left(x^2 - \frac{7}{2}x\right)$ $= 2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right]$ $= 2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8}$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$ **2** Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$ **3** Expand and Simplify

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Practice questions

1 Complete the square for the following expressions:

a $x^2 + 8x$ **b** $x^2 - 10x$ **c** $x^2 - x$

d $3x^2 - 15x$ **e** $12x - 2x^2$

Answers

1 Solve by completing the square.

a $(x+4)^2 - 16$ **b** $(x-5)^2 - 25$ **c** $\left(x - \frac{1}{2}\right)^2 - \frac{1}{4}$

d $3\left(x - \frac{5}{2}\right)^2 - \frac{75}{4}$ **e** $-2(x-3)^2 + 18$

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Exercise 8: Solving linear simultaneous equations by elimination

A LEVEL LINKS **Scheme of work:** 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$3x + y = 5$ $-\quad x + y = 1$ $2x \quad = 4$ So $x = 2$	**1** Subtract the second equation from the first equation to eliminate the y term.
Using $x + y = 1$ $2 + y = 1$ So $y = -1$	**2** To find the value of y, substitute $x = 2$ into one of the original equations.
Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$x + 2y = 13$ $+ \quad 5x - 2y = 5$ $6x \quad = 18$ So $x = 3$	**1** Add the two equations together to eliminate the y term.
Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$	**2** To find the value of y, substitute $x = 3$ into one of the original equations.
Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Practice questions

Solve these simultaneous equations.

1 $4x + y = 8$
$x + y = 5$

2 $3x + y = 7$
$3x + 2y = 5$

3 $4x + y = 3$
$3x - y = 11$

4 $3x + 4y = 7$
$x - 4y = 5$

5 $2x + y = 11$
$x - 3y = 9$

6 $2x + 3y = 11$
$3x + 2y = 4$

7 $4x + y = 25$
$x - 3y = 16$

Answers

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$

6 $x = -2, y = 5$

7 $x = 7, y = -3$