DIXONS SIXTH FORM ACADEMY

SUMMER WORK A LEVEL MATHS

2024

STUDENT NAME:

Contents

About the Summer Work

A Level Mathematics takes as a base your knowledge of topics from GCSE. The following topics will be tested on arrival in September. Very few lessons will be spent going over these basic principles.

Time should be spent over the summer months preparing for the preliminary test on the first week back.

Guidance

- Complete these tasks on A4 paper and bring them with you to your first maths lesson.
- Each page should be labelled with the title of the task and question numbers included.
- You should staple all the Algebra tasks together; all the Coordinate Geometry tasks together and all the trigonometry tasks together.
- Work should be self-marked using green pen and corrected for errors.
- Attempt every question and always show your working.
- Spend additional time on tasks you struggle with, using corbettmaths videos to help you where necessary.
- This booklet also contains significant additional information. We would encourage you to complete all the tasks including the optional ones to fully prepare for Sixth Form study.
- Use the week-by-week schedule as a guide to how much you should be aiming to complete each week.

Algebra

1. Expanding Brackets and Simplifying Expressions
2. Surds and Rationalising Surds
3. Rules of Indices
4. Factorising Expressions
5. Completing the Square
6. Rearranging Equations
7. Solving Quadratics (Factorisation, Completing the Square, Quadratic Formula)
8. Solving Simultaneous Equations
9. Linear Inequalities
10. Graph Transformations

Coordinate Geometry

11. Straight Line Graphs
12. Parallel and Perpendicular Lines

Trigonometry

13. Trigonometry in Right Angled Triangles
14. The Cosine Rule
15. The Sine Rule
16. Area of Triangles

Welcome to Mathematics

Subject outline

A-level maths is the most popular A-level taken by students who go on to university. The subject sharpens many key skills, including the ability to get to grips with problems, something that lies at the centre of many fields. Students who study maths at A-Level relish a challenge and enjoy investigating different processes. Successful students understand the power of practice in mathematics and invest their study time in completing exercises and past exam questions frequently to become familiar with a range of different contexts. Outside of lesson time, students need to use their independent study time to practice maths and check through solutions from a range of resources including online retrieval practice exercises, textbook procedural practice and past exam papers.

Mathematics has always been a highly valued A-Level by Universities and employers due to its complex content and the demands of the course. Mathematics opens pathways for students to a wide range of courses that require students to be highly numerate and strong logical thinkers. In our technology focused society, mathematics students can often show innovation and creativity in approaching a challenge and working to find a solution, traits which are essential in the modern-day work force.

Students will study the Edexcel Specification for A Level Mathematics, with three 2-hour papers at the end of year 2.

Paper 1&2 – Pure Mathematics (concepts such as algebra, coordinate geometry, graphs and calculus)

Paper 3 – Mechanics and Statistics (including forces, kinematics and projectiles for mechanics and probability distributions, data handling, hypothesis testing in statistics)

Careers & Higher Education

A Level Mathematics is a facilitating subject, meaning that is a highly respected A Level qualification that is listed as essential on some university courses. If you are interested in studying engineering, economics, mathematics, physics, statistics, actuarial science or computer science most universities require you to complete A Level Mathematics. If you are interested in other routes such as biochemistry, dentistry, business studies, geography or accounting some universities may list mathematics as a useful subject but not essential.

Maths careers are some of the most highly paid careers available. Research shows that on average A Level maths students earn 11% more than other students during their lifetime. Many believe that taking maths at university has limited fields since it doesn't move straight into a vocation. However, this is certainly not the case. Students who continue maths at university can move into various careers, from graduate roles within the finance industry to working in a graduate role within the civil service. Engineering has many different degree routes and courses and is one of the most popular areas that students choose to work in after university.

An example of a highly mathematical career is an actuary. An actuary works in a business analysing risk, often within the financial sector. Actuaries use mathematical modelling techniques and statistical concepts to determine probability and assess risks, for example, analysing pension scheme liabilities to price commercial insurance. Due to the challenging nature of the exams required to become a qualified actuary, the salary is usually very competitive.

Links to key information:

dixons6a.com/uploads/files/Maths.pdf

qualifications.pearson.com/en/qualifications/edexcel-a-levels/mathematics-2017.html

https://amsp.org.uk/teachers/11-16-maths/transition-to-level-3-maths/where-maths-meets-the-world-of-work/

Summer work tasks

Weekly Tasks

There are 16 practice tasks to complete, with examples and answers, that are all revision of key topics from GCSE. These topics have been split into three overarching units: Algebra, Coordinate Geometry and Trigonometry. These topics are essential to the study of A Level Mathematics and students need to ensure they fully understand each concept. **Students will be assessed in the first lesson on these topics** to ensure they are starting the course with a grounded understanding of algebra from GCSE. Please see the guidance on the previous page on how to set out your work. Below is a suggested week-by-week schedule to help you organize your time.

Week	Unit	Exercise
1	Algebra	1. Expanding Brackets and Simplifying Expressions 2. Surds and Rationalising Surds 3. Rules of indices 4. Factorising Expressions
2	Algebra	1. Completing the Square 2. Rearranging Equations 3. Solving Quadratics
3	Algebra	1. Solving Simultaneous Equations (Elimination and Substitution) 2. Linear Inequalities 3. Graph Transformations
4	Coordinate Geometry	1. Straight Line Graphs 2. Parallel and Perpendicular Lines
5	Trigonometry	1. Trigonometry in Right Angled Triangles 2. The Cosine Rule
6	Trigonometry	1. The Sine Rule 2. Area of Triangles

Reading list

Suggested reading:

The Codebook by *Simon Singh*

The Simpsons and Their Mathematical Secrets by *Simon Singh*

Infinity: The Quest to Think the Unthinkable by *Brian Clegg*

The Man who knew Infinity by *Robert Kanigel*

Humble Pi: A Comedy of Maths Errors by *Matt Parker*

Suggested viewing:

bbc.co.uk/iplayer/episode/b0074rxx/horizon-19951996-fermats-last-theorem

Expanding brackets and simplifying expressions

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- When you expand one set of brackets you must multiply everything inside the bracket by what is outside.
- When you expand two linear expressions, each with two terms of the form $ax + b$, where $a \neq 0$ and $b \neq 0$, you create four terms. Two of these can usually be simplified by collecting like terms.

Examples

Example 1 Expand $4(3x - 2)$

$4(3x - 2) = 12x - 8$	Multiply everything inside the bracket by the 4 outside the bracket

Example 2 Expand and simplify $3(x + 5) - 4(2x + 3)$

$3(x + 5) - 4(2x + 3)$ $= 3x + 15 - 8x - 12$ $= 3 - 5x$	**1** Expand each set of brackets separately by multiplying $(x + 5)$ by 3 and $(2x + 3)$ by -4 **2** Simplify by collecting like terms: $3x - 8x = -5x$ and $15 - 12 = 3$

Example 3 Expand and simplify $(x + 3)(x + 2)$

$(x + 3)(x + 2)$ $= x(x + 2) + 3(x + 2)$ $= x^2 + 2x + 3x + 6$ $= x^2 + 5x + 6$	**1** Expand the brackets by multiplying $(x + 2)$ by x and $(x + 2)$ by 3 **2** Simplify by collecting like terms: $2x + 3x = 5x$

Example 4 Expand and simplify $(x - 5)(2x + 3)$

$(x - 5)(2x + 3)$ $= x(2x + 3) - 5(2x + 3)$ $= 2x^2 + 3x - 10x - 15$ $= 2x^2 - 7x - 15$	**1** Expand the brackets by multiplying $(2x + 3)$ by x and $(2x + 3)$ by -5 **2** Simplify by collecting like terms: $3x - 10x = -7x$

Practice

Watch out!
When multiplying (or dividing) positive and negative numbers, if the signs are the same the answer is '+'; if the signs are different the answer is '–'.

1 Expand.

a $3(2x - 1)$ **b** $-2(5pq + 4q^2)$

c $-(3xy - 2y^2)$

2 Expand and simplify.

a $7(3x + 5) + 6(2x - 8)$ **b** $8(5p - 2) - 3(4p + 9)$

c $9(3s + 1) - 5(6s - 10)$ **d** $2(4x - 3) - (3x + 5)$

3 Expand.

a $3x(4x + 8)$ **b** $4k(5k^2 - 12)$

c $-2h(6h^2 + 11h - 5)$ **d** $-3s(4s^2 - 7s + 2)$

4 Expand and simplify.

a $3(y^2 - 8) - 4(y^2 - 5)$ **b** $2x(x + 5) + 3x(x - 7)$

c $4p(2p - 1) - 3p(5p - 2)$ **d** $3b(4b - 3) - b(6b - 9)$

5 Expand $\frac{1}{2}(2y - 8)$

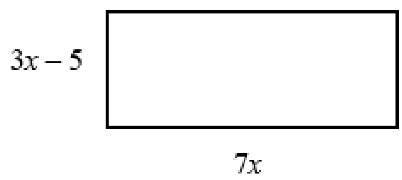
6 Expand and simplify.

a $13 - 2(m + 7)$ **b** $5p(p^2 + 6p) - 9p(2p - 3)$

7 The diagram shows a rectangle.

Write down an expression, in terms of x, for the area of the rectangle.

Show that the area of the rectangle can be written as $21x^2 - 35x$



8 Expand and simplify.

a $(x + 4)(x + 5)$ **b** $(x + 7)(x + 3)$

c $(x + 7)(x - 2)$ **d** $(x + 5)(x - 5)$

e $(2x + 3)(x - 1)$ **f** $(3x - 2)(2x + 1)$

g $(5x - 3)(2x - 5)$ **h** $(3x - 2)(7 + 4x)$

i $(3x + 4y)(5y + 6x)$ **j** $(x + 5)^2$

k $(2x - 7)^2$ **l** $(4x - 3y)^2$

Extend

9 Expand and simplify $(x + 3)^2 + (x - 4)^2$

10 Expand and simplify.

a $\left(x+\frac{1}{x}\right)\left(x-\frac{2}{x}\right)$ **b** $\left(x+\frac{1}{x}\right)^2$

Answers

1 **a** $6x - 3$ **b** $-10pq - 8q^2$
c $-3xy + 2y^2$

2 **a** $21x + 35 + 12x - 48 = 33x - 13$
b $40p - 16 - 12p - 27 = 28p - 43$
c $27s + 9 - 30s + 50 = -3s + 59 = 59 - 3s$
d $8x - 6 - 3x - 5 = 5x - 11$

3 **a** $12x^2 + 24x$ **b** $20k^3 - 48k$
c $10h - 12h^3 - 22h^2$ **d** $21s^2 - 21s^3 - 6s$

4 **a** $-y^2 - 4$ **b** $5x^2 - 11x$
c $2p - 7p^2$ **d** $6b^2$

5 $y - 4$

6 **a** $-1 - 2m$ **b** $5p^3 + 12p^2 + 27p$

7 $7x(3x - 5) = 21x^2 - 35x$

8 **a** $x^2 + 9x + 20$ **b** $x^2 + 10x + 21$
c $x^2 + 5x - 14$ **d** $x^2 - 25$
e $2x^2 + x - 3$ **f** $6x^2 - x - 2$
g $10x^2 - 31x + 15$ **h** $12x^2 + 13x - 14$
i $18x^2 + 39xy + 20y^2$ **j** $x^2 + 10x + 25$
k $4x^2 - 28x + 49$ **l** $16x^2 - 24xy + 9y^2$

9 $2x^2 - 2x + 25$

10 **a** $x^2 - 1 - \dfrac{2}{x^2}$ **b** $x^2 + 2 + \dfrac{1}{x^2}$

Surds and rationalising the denominator

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}, \sqrt{3}, \sqrt{5}$, etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd $\sqrt{b}$
- To rationalise $\frac{a}{b+\sqrt{c}}$ you multiply the numerator and denominator by $b-\sqrt{c}$

Examples

Example 1 Simplify $\sqrt{50}$

$\sqrt{50} = \sqrt{25 \times 2}$	**1** Choose two numbers that are factors of 50. One of the factors must be a square number
$= \sqrt{25} \times \sqrt{2}$	**2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= 5 \times \sqrt{2}$	**3** Use $\sqrt{25} = 5$
$= 5\sqrt{2}$	

Example 2 Simplify $\sqrt{147} - 2\sqrt{12}$

$\sqrt{147} - 2\sqrt{12}$ $= \sqrt{49 \times 3} - 2\sqrt{4 \times 3}$	**1** Simplify $\sqrt{147}$ and $2\sqrt{12}$. Choose two numbers that are factors of 147 and two numbers that are factors of 12. One of each pair of factors must be a square number **2** Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
$= \sqrt{49} \times \sqrt{3} - 2\sqrt{4} \times \sqrt{3}$	**3** Use $\sqrt{49} = 7$ and $\sqrt{4} = 2$
$= 7 \times \sqrt{3} - 2 \times 2 \times \sqrt{3}$ $= 7\sqrt{3} - 4\sqrt{3}$ $= 3\sqrt{3}$	**4** Collect like terms

Example 3 Simplify $\left(\sqrt{7} + \sqrt{2}\right)\left(\sqrt{7} - \sqrt{2}\right)$

$\left(\sqrt{7} + \sqrt{2}\right)\left(\sqrt{7} - \sqrt{2}\right)$ $= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4}$	**1** Expand the brackets. A common mistake here is to write $\left(\sqrt{7}\right)^2 = 49$
$= 7 - 2$ $= 5$	**2** Collect like terms: $-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7}$ $= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0$

Example 4 Rationalise $\frac{1}{\sqrt{3}}$

$\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$	**1** Multiply the numerator and denominator by $\sqrt{3}$
$= \frac{1 \times \sqrt{3}}{\sqrt{9}}$	**2** Use $\sqrt{9} = 3$
$= \frac{\sqrt{3}}{3}$	

Example 5 Rationalise and simplify $\frac{\sqrt{2}}{\sqrt{12}}$

$\frac{\sqrt{2}}{\sqrt{12}} = \frac{\sqrt{2}}{\sqrt{12}} \times \frac{\sqrt{12}}{\sqrt{12}}$	**1** Multiply the numerator and denominator by $\sqrt{12}$
$= \frac{\sqrt{2} \times \sqrt{4 \times 3}}{12}$	**2** Simplify $\sqrt{12}$ in the numerator. Choose two numbers that are factors of 12. One of the factors must be a square number
	3 Use the rule $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
	4 Use $\sqrt{4} = 2$
$= \frac{2\sqrt{2}\sqrt{3}}{12}$	**5** Simplify the fraction: $\frac{2}{12}$ simplifies to $\frac{1}{6}$
$= \frac{\sqrt{2}\sqrt{3}}{6}$	

Example 6 Rationalise and simplify $\frac{3}{2+\sqrt{5}}$

$\frac{3}{2+\sqrt{5}} = \frac{3}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}}$	**1** Multiply the numerator and denominator by $2-\sqrt{5}$
$= \frac{3\left(2-\sqrt{5}\right)}{\left(2+\sqrt{5}\right)\left(2-\sqrt{5}\right)}$	
$= \frac{6-3\sqrt{5}}{4+2\sqrt{5}-2\sqrt{5}-5}$	**2** Expand the brackets
$= \frac{6-3\sqrt{5}}{-1}$	**3** Simplify the fraction
$= 3\sqrt{5}-6$	**4** Divide the numerator by −1 Remember to change the sign of all terms when dividing by −1

Practice

1 Simplify.

a $\sqrt{45}$ **b** $\sqrt{125}$

c $\sqrt{48}$ **d** $\sqrt{175}$

e $\sqrt{300}$ **f** $\sqrt{28}$

g $\sqrt{72}$ **h** $\sqrt{162}$

Hint
One of the two numbers you choose at the start must be a square number.

2 Simplify.

a $\sqrt{72}+\sqrt{162}$ **b** $\sqrt{45}-2\sqrt{5}$

c $\sqrt{50}-\sqrt{8}$ **d** $\sqrt{75}-\sqrt{48}$

e $2\sqrt{28}+\sqrt{28}$ **f** $2\sqrt{12}-\sqrt{12}+\sqrt{27}$

Watch out!
Check you have chosen the highest square number at the start.

3 Expand and simplify.

a $(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})$ **b** $(3+\sqrt{3})(5-\sqrt{12})$

c $(4-\sqrt{5})(\sqrt{45}+2)$ **d** $(5+\sqrt{2})(6-\sqrt{8})$

4 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{5}}$ **b** $\frac{1}{\sqrt{11}}$

c $\frac{2}{\sqrt{7}}$ **d** $\frac{2}{\sqrt{8}}$

e $\frac{2}{\sqrt{2}}$ **f** $\frac{5}{\sqrt{5}}$

g $\frac{\sqrt{8}}{\sqrt{24}}$ **h** $\frac{\sqrt{5}}{\sqrt{45}}$

5 Rationalise and simplify.

a $\frac{1}{3-\sqrt{5}}$ **b** $\frac{2}{4+\sqrt{3}}$ **c** $\frac{6}{5-\sqrt{2}}$

Extend

6 Expand and simplify $\left(\sqrt{x}+\sqrt{y}\right)\left(\sqrt{x}-\sqrt{y}\right)$

7 Rationalise and simplify, if possible.

a $\frac{1}{\sqrt{9}-\sqrt{8}}$ **b** $\frac{1}{\sqrt{x}-\sqrt{y}}$

Answers

1	**a**	$3\sqrt{5}$	**b**	$5\sqrt{5}$		
	c	$4\sqrt{3}$	**d**	$5\sqrt{7}$		
	e	$10\sqrt{3}$	**f**	$2\sqrt{7}$		
	g	$6\sqrt{2}$	**h**	$9\sqrt{2}$		
2	**a**	$15\sqrt{2}$	**b**	$\sqrt{5}$		
	c	$3\sqrt{2}$	**d**	$\sqrt{3}$		
	e	$6\sqrt{7}$	**f**	$5\sqrt{3}$		
3	**a**	-1	**b**	$9-\sqrt{3}$		
	c	$10\sqrt{5}-7$	**d**	$26-4\sqrt{2}$		
4	**a**	$\frac{\sqrt{5}}{5}$	**b**	$\frac{\sqrt{11}}{11}$		
	c	$\frac{2\sqrt{7}}{7}$	**d**	$\frac{\sqrt{2}}{2}$		
	e	$\sqrt{2}$	**f**	$\sqrt{5}$		
	g	$\frac{\sqrt{3}}{3}$	**h**	$\frac{1}{3}$		
5	**a**	$\frac{3+\sqrt{5}}{4}$	**b**	$\frac{2(4-\sqrt{3})}{13}$	**c**	$\frac{6(5+\sqrt{2})}{23}$
6	$x-y$					
7	**a**	$3+2\sqrt{2}$	**b**	$\frac{\sqrt{x}+\sqrt{y}}{x-y}$		

Rules of indices

A LEVEL LINKS
Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\dfrac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the *n*th root of *a*
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = \left(\sqrt[n]{a}\right)^m$
- $a^{-m} = \dfrac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Examples

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1

Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$

Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = \left(\sqrt[3]{27}\right)^2$ $= 3^2$ $= 9$	**1** Use the rule $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ **2** Use $\sqrt[3]{27} = 3$

Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	**1** Use the rule $a^{-m} = \frac{1}{a^m}$ **2** Use $4^2 = 16$

Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$

Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	**1** Use the rule $a^m \times a^n = a^{m+n}$ **2** Use the rule $\frac{a^m}{a^n} = a^{m-n}$

Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged

Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	**1** Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ **2** Use the rule $\frac{1}{a^m} = a^{-m}$

Practice

1 Evaluate.

a 14^0 **b** 3^0 **c** 5^0 **d** x^0

2 Evaluate.

a $49^{\frac{1}{2}}$ **b** $64^{\frac{1}{3}}$ **c** $125^{\frac{1}{3}}$ **d** $16^{\frac{1}{4}}$

3 Evaluate.

a $25^{\frac{3}{2}}$ **b** $8^{\frac{5}{3}}$ **c** $49^{\frac{3}{2}}$ **d** $16^{\frac{3}{4}}$

4 Evaluate.

a 5^{-2} **b** 4^{-3} **c** 2^{-5} **d** 6^{-2}

5 Simplify.

a $\dfrac{3x^2 \times x^3}{2x^2}$ **b** $\dfrac{10x^5}{2x^2 \times x}$

c $\dfrac{3x \times 2x^3}{2x^3}$ **d** $\dfrac{7x^3y^2}{14x^5y}$

e $\dfrac{y^2}{y^{\frac{1}{2}} \times y}$ **f** $\dfrac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g $\dfrac{(2x^2)^3}{4x^0}$ **h** $\dfrac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

Watch out!
Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

6 Evaluate.

a $4^{-\frac{1}{2}}$ **b** $27^{-\frac{2}{3}}$ **c** $9^{-\frac{1}{2}} \times 2^3$

d $16^{\frac{1}{4}} \times 2^{-3}$ **e** $\left(\dfrac{9}{16}\right)^{-\frac{1}{2}}$ **f** $\left(\dfrac{27}{64}\right)^{-\frac{2}{3}}$

7 Write the following as a single power of x.

a $\dfrac{1}{x}$ **b** $\dfrac{1}{x^7}$ **c** $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$ **e** $\dfrac{1}{\sqrt[3]{x}}$ **f** $\dfrac{1}{\sqrt[3]{x^2}}$

8 Write the following without negative or fractional powers.

a x^{-3} **b** x^0 **c** $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$ **e** $x^{-\frac{1}{2}}$ **f** $x^{-\frac{3}{4}}$

9 Write the following in the form ax^n.

a $5\sqrt{x}$ **b** $\frac{2}{x^3}$ **c** $\frac{1}{3x^4}$

d $\frac{2}{\sqrt{x}}$ **e** $\frac{4}{\sqrt[3]{x}}$ **f** 3

Extend

10 Write as sums of powers of x.

a $\frac{x^5+1}{x^2}$ **b** $x^2\left(x+\frac{1}{x}\right)$ **c** $x^{-4}\left(x^2+\frac{1}{x^3}\right)$

Answers

1	**a**	1	**b**	1	**c**	1	**d**	1
2	**a**	7	**b**	4	**c**	5	**d**	2
3	**a**	125	**b**	32	**c**	343	**d**	8
4	**a**	$\frac{1}{25}$	**b**	$\frac{1}{64}$	**c**	$\frac{1}{32}$	**d**	$\frac{1}{36}$
5	**a**	$\frac{3x^3}{2}$	**b**	$5x^2$				
	c	$3x$	**d**	$\frac{y}{2x^2}$				
	e	$y^{\frac{1}{2}}$	**f**	c^{-3}				
	g	$2x^6$	**h**	x				
6	**a**	$\frac{1}{2}$	**b**	$\frac{1}{9}$	**c**	$\frac{8}{3}$		
	d	$\frac{1}{4}$	**e**	$\frac{4}{3}$	**f**	$\frac{16}{9}$		
7	**a**	x^{-1}	**b**	x^{-7}	**c**	$x^{\frac{1}{4}}$		
	d	$x^{\frac{2}{5}}$	**e**	$x^{-\frac{1}{3}}$	**f**	$x^{-\frac{2}{3}}$		
8	**a**	$\frac{1}{x^3}$	**b**	1	**c**	$\sqrt[5]{x}$		
	d	$\sqrt[5]{x^2}$	**e**	$\frac{1}{\sqrt{x}}$	**f**	$\frac{1}{\sqrt[4]{x^3}}$		
9	**a**	$5x^{\frac{1}{2}}$	**b**	$2x^{-3}$	**c**	$\frac{1}{3}x^{-4}$		
	d	$2x^{-\frac{1}{2}}$	**e**	$4x^{-\frac{1}{3}}$	**f**	$3x^0$		
10	**a**	$x^3 + x^{-2}$	**b**	$x^3 + x$	**c**	$x^{-2} + x^{-7}$		

Factorising expressions

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Factorising an expression is the opposite of expanding the brackets.
- A quadratic expression is in the form $ax^2 + bx + c$, where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose product is ac.
- An expression in the form $x^2 - y^2$ is called the difference of two squares. It factorises to $(x - y)(x + y)$.

Examples

Example 1 Factorise $15x^2y^3 + 9x^4y$

$15x^2y^3 + 9x^4y = 3x^2y(5y^2 + 3x^2)$	The highest common factor is $3x^2y$. So take $3x^2y$ outside the brackets and then divide each term by $3x^2y$ to find the terms in the brackets

Example 2 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$

Example 3 Factorise $x^2 + 3x - 10$

$b = 3$, $ac = -10$	**1** Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and −2)
So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$	**2** Rewrite the b term ($3x$) using these two factors
$= x(x + 5) - 2(x + 5)$	**3** Factorise the first two terms and the last two terms
$= (x + 5)(x - 2)$	**4** $(x + 5)$ is a factor of both terms

Example 4 Factorise $6x^2 - 11x - 10$

$b = -11$, $ac = -60$ So $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	**1** Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) **2** Rewrite the b term ($-11x$) using these two factors **3** Factorise the first two terms and the last two terms **4** $(2x - 5)$ is a factor of both terms

Example 5 Simplify $\frac{x^2-4x-21}{2x^2+9x+9}$

$\frac{x^2-4x-21}{2x^2+9x+9}$	**1** Factorise the numerator and the denominator
For the numerator: $b = -4$, $ac = -21$ So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$	**2** Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3) **3** Rewrite the b term ($-4x$) using these two factors **4** Factorise the first two terms and the last two terms **5** $(x - 7)$ is a factor of both terms
For the denominator: $b = 9$, $ac = 18$ So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$	**6** Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3) **7** Rewrite the b term ($9x$) using these two factors **8** Factorise the first two terms and the last two terms **9** $(x + 3)$ is a factor of both terms **10** $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1

$= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$ So $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x-7)(x+3)}{(x+3)(2x+3)}$ $= \frac{x-7}{2x+3}$	

Practice

Hint
Take the highest common factor outside the bracket.

1 Factorise.

a $6x^4y^3 - 10x^3y^4$ **b** $21a^3b^5 + 35a^5b^2$

c $25x^2y^2 - 10x^3y^2 + 15x^2y^3$

2 Factorise

a $x^2 + 7x + 12$ **b** $x^2 + 5x - 14$

c $x^2 - 11x + 30$ **d** $x^2 - 5x - 24$

e $x^2 - 7x - 18$ **f** $x^2 + x - 20$

g $x^2 - 3x - 40$ **h** $x^2 + 3x - 28$

3 Factorise

a $36x^2 - 49y^2$ **b** $4x^2 - 81y^2$

c $18a^2 - 200b^2c^2$

4 Factorise

a $2x^2 + x - 3$ **b** $6x^2 + 17x + 5$

c $2x^2 + 7x + 3$ **d** $9x^2 - 15x + 4$

e $10x^2 + 21x + 9$ **f** $12x^2 - 38x + 20$

5 Simplify the algebraic fractions.

a $\dfrac{2x^2+4x}{x^2-x}$ **b** $\dfrac{x^2+3x}{x^2+2x-3}$

c $\dfrac{x^2-2x-8}{x^2-4x}$ **d** $\dfrac{x^2-5x}{x^2-25}$

e $\dfrac{x^2-x-12}{x^2-4x}$ **f** $\dfrac{2x^2+14x}{2x^2+4x-70}$

6 Simplify

a $\dfrac{9x^2-16}{3x^2+17x-28}$ **b** $\dfrac{2x^2-7x-15}{3x^2-17x+10}$

c $\dfrac{4-25x^2}{10x^2-11x-6}$ **d** $\dfrac{6x^2-x-1}{2x^2+7x-4}$

Extend

7 Simplify $\sqrt{x^2+10x+25}$

8 Simplify $\dfrac{(x+2)^2+3(x+2)^2}{x^2-4}$

Answers

1 **a** $2x^3y^3(3x - 5y)$ **b** $7a^3b^2(3b^3 + 5a^2)$
c $5x^2y^2(5 - 2x + 3y)$

2 **a** $(x + 3)(x + 4)$ **b** $(x + 7)(x - 2)$
c $(x - 5)(x - 6)$ **d** $(x - 8)(x + 3)$
e $(x - 9)(x + 2)$ **f** $(x + 5)(x - 4)$
g $(x - 8)(x + 5)$ **h** $(x + 7)(x - 4)$

3 **a** $(6x - 7y)(6x + 7y)$ **b** $(2x - 9y)(2x + 9y)$
c $2(3a - 10bc)(3a + 10bc)$

4 **a** $(x - 1)(2x + 3)$ **b** $(3x + 1)(2x + 5)$
c $(2x + 1)(x + 3)$ **d** $(3x - 1)(3x - 4)$
e $(5x + 3)(2x + 3)$ **f** $2(3x - 2)(2x - 5)$

5 **a** $\frac{2(x+2)}{x-1}$ **b** $\frac{x}{x-1}$
c $\frac{x+2}{x}$ **d** $\frac{x}{x+5}$
e $\frac{x+3}{x}$ **f** $\frac{x}{x-5}$

6 **a** $\frac{3x+4}{x+7}$ **b** $\frac{2x+3}{3x-2}$
c $\frac{2-5x}{2x-3}$ **d** $\frac{3x+1}{x+4}$

7 $(x + 5)$

8 $\frac{4(x+2)}{x-2}$

Completing the square

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square for a quadratic rearranges $ax^2 + bx + c$ into the form $p(x + q)^2 + r$
- If $a \neq 1$, then factorise using a as a common factor.

Examples

Example 1 Complete the square for the quadratic expression $x^2 + 6x - 2$

$x^2 + 6x - 2$ $= (x + 3)^2 - 9 - 2$ $= (x + 3)^2 - 11$	**1** Write $x^2 + bx + c$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$ **2** Simplify

Example 2 Write $2x^2 - 5x + 1$ in the form $p(x + q)^2 + r$

$2x^2 - 5x + 1$ $= 2\left(x^2 - \frac{5}{2}x\right) + 1$ $= 2\left[\left(x - \frac{5}{4}\right)^2 - \left(\frac{5}{4}\right)^2\right] + 1$ $= 2\left(x - \frac{5}{4}\right)^2 - \frac{25}{8} + 1$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$ **2** Now complete the square by writing $x^2 - \frac{5}{2}x$ in the form $\left(x+\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$ **3** Expand the square brackets – don't forget to multiply $\left(\frac{5}{4}\right)^2$ by the factor of 2 **4** Simplify

$= 2\left(x - \frac{5}{4}\right)^2 - \frac{17}{8}$	

Practice

1 Write the following quadratic expressions in the form $(x + p)^2 + q$

a $x^2 + 4x + 3$ **b** $x^2 - 10x - 3$

c $x^2 - 8x$ **d** $x^2 + 6x$

e $x^2 - 2x + 7$ **f** $x^2 + 3x - 2$

2 Write the following quadratic expressions in the form $p(x + q)^2 + r$

a $2x^2 - 8x - 16$ **b** $4x^2 - 8x - 16$

c $3x^2 + 12x - 9$ **d** $2x^2 + 6x - 8$

3 Complete the square.

a $2x^2 + 3x + 6$ **b** $3x^2 - 2x$

c $5x^2 + 3x$ **d** $3x^2 + 5x + 3$

Extend

4 Write $(25x^2 + 30x + 12)$ in the form $(ax + b)^2 + c$.

Answers

1 **a** $(x+2)^2-1$ **b** $(x-5)^2-28$

c $(x-4)^2-16$ **d** $(x+3)^2-9$

e $(x-1)^2+6$ **f** $\left(x+\frac{3}{2}\right)^2-\frac{17}{4}$

2 **a** $2(x-2)^2-24$ **b** $4(x-1)^2-20$

c $3(x+2)^2-21$ **d** $2\left(x+\frac{3}{2}\right)^2-\frac{25}{2}$

3 **a** $2\left(x+\frac{3}{4}\right)^2+\frac{39}{8}$ **b** $3\left(x-\frac{1}{3}\right)^2-\frac{1}{3}$

c $5\left(x+\frac{3}{10}\right)^2-\frac{9}{20}$ **d** $3\left(x+\frac{5}{6}\right)^2+\frac{11}{12}$

4 $(5x+3)^2+3$

Rearranging equations

A LEVEL LINKS
Scheme of work: 6a. Definition, differentiating polynomials, second derivatives
Textbook: Pure Year 1, 12.1 Gradients of curves

Key points

- To change the subject of a formula, get the terms containing the subject on one side and everything else on the other side.
- You may need to factorise the terms containing the new subject.

Examples

Example 1 Make t the subject of the formula $v = u + at$.

$v = u + at$	**1** Get the terms containing t on one side and everything else on the other side.
$v - u = at$	**2** Divide throughout by a.
$t = \frac{v-u}{a}$	

Example 2 Make t the subject of the formula $r = 2t - \pi t$.

$r = 2t - \pi t$	**1** All the terms containing t are already on one side and everything else is on the other side.
	2 Factorise as t is a common factor.
$r = t(2 - \pi)$	**3** Divide throughout by $2 - \pi$.
$t = \frac{r}{2-\pi}$	

Example 3 Make t the subject of the formula $\frac{t+r}{5} = \frac{3t}{2}$.

$\frac{t+r}{5} = \frac{3t}{2}$	**1** Remove the fractions first by multiplying throughout by 10.
$2t + 2r = 15t$	**2** Get the terms containing t on one side and everything else on the other side and simplify.
$2r = 13t$	**3** Divide throughout by 13.

$t = \frac{2r}{13}$	

Example 4 Make t the subject of the formula $r = \frac{3t+5}{t-1}$.

$r = \frac{3t+5}{t-1}$ $r(t-1) = 3t + 5$ $rt - r = 3t + 5$ $rt - 3t = 5 + r$ $t(r-3) = 5 + r$ $t = \frac{5+r}{r-3}$	**1** Remove the fraction first by multiplying throughout by $t - 1$. **2** Expand the brackets. **3** Get the terms containing t on one side and everything else on the other side. **4** Factorise the LHS as t is a common factor. **5** Divide throughout by $r - 3$.

Practice

Change the subject of each formula to the letter given in the brackets.

1 $C = \pi d$ [d]

2 $P = 2l + 2w$ [w]

3 $D = \frac{S}{T}$ [T]

4 $p = \frac{q-r}{t}$ [t]

5 $u = at - \frac{1}{2}t$ [t]

6 $V = ax + 4x$ [x]

7 $\frac{y-7x}{2} = \frac{7-2y}{3}$ [y]

8 $x = \frac{2a-1}{3-a}$ [a]

9 $x = \frac{b-c}{d}$ [d]

10 $h = \frac{7g-9}{2+g}$ [g]

11 $e(9 + x) = 2e + 1$ [e]

12 $y = \frac{2x+3}{4-x}$ [x]

13 Make r the subject of the following formulae.

a $A = \pi r^2$ **b** $V = \frac{4}{3}\pi r^3$ **c** $P = \pi r + 2r$ **d** $V = \frac{2}{3}\pi r^2 h$

14 Make x the subject of the following formulae.

a $\frac{xy}{z} = \frac{ab}{cd}$ **b** $\frac{4\pi cx}{d} = \frac{3z}{py^2}$

15 Make $\sin B$ the subject of the formula $\frac{a}{\sin A} = \frac{b}{\sin B}$

16 Make $\cos B$ the subject of the formula $b^2 = a^2 + c^2 - 2ac \cos B$.

Extend

17 Make x the subject of the following equations.

a $\frac{p}{q}(sx+t)=x-1$

b $\frac{p}{q}(ax+2y)=\frac{3p}{q^2}(x-y)$

Answers

1 $d = \dfrac{C}{\pi}$ **2** $w = \dfrac{P - 2l}{2}$ **3** $T = \dfrac{S}{D}$

4 $t = \dfrac{q - r}{p}$ **5** $t = \dfrac{2u}{2a - 1}$ **6** $x = \dfrac{V}{a + 4}$

7 $y = 2 + 3x$ **8** $a = \dfrac{3x + 1}{x + 2}$ **9** $d = \dfrac{b - c}{x}$

10 $g = \dfrac{2h + 9}{7 - h}$ **11** $e = \dfrac{1}{x + 7}$ **12** $x = \dfrac{4y - 3}{2 + y}$

13 **a** $r = \sqrt{\dfrac{A}{\pi}}$ **b** $r = \sqrt[3]{\dfrac{3V}{4\pi}}$

c $r = \dfrac{P}{\pi + 2}$ **d** $r = \sqrt{\dfrac{3V}{2\pi h}}$

14 **a** $x = \dfrac{abz}{cdy}$ **b** $x = \dfrac{3dz}{4\pi cpy^2}$

15 $\sin B = \dfrac{b \sin A}{a}$

16 $\cos B = \dfrac{a^2 + c^2 - b^2}{2ac}$

17 **a** $x = \dfrac{q + pt}{q - ps}$ **b** $x = \dfrac{3py + 2pqy}{3p - apq} = \dfrac{y(3 + 2q)}{3 - aq}$

Solving quadratic equations by factorisation

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac.
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Examples

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$ $5x(x - 3) = 0$ So $5x = 0$ or $(x - 3) = 0$ Therefore $x = 0$ or $x = 3$	**1** Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$. **2** Factorise the quadratic equation. $5x$ is a common factor. **3** When two values multiply to make zero, at least one of the values must be zero. **4** Solve these two equations.

Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7$, $ac = 12$ $x^2 + 4x + 3x + 12 = 0$ $x(x + 4) + 3(x + 4) = 0$ $(x + 4)(x + 3) = 0$ So $(x + 4) = 0$ or $(x + 3) = 0$ Therefore $x = −4$ or $x = −3$	**1** Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3) **2** Rewrite the b term ($7x$) using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x + 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ So $(3x + 4) = 0$ or $(3x - 4) = 0$ $x = -\frac{4}{3}$ or $x = \frac{4}{3}$	**1** Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. **2** When two values multiply to make zero, at least one of the values must be zero. **3** Solve these two equations.

Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5$, $ac = -24$ So $2x^2 - 8x + 3x - 12 = 0$ $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ So $(x - 4) = 0$ or $(2x + 3) = 0$ $x = 4$ or $x = -\frac{3}{2}$	**1** Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. $(-8$ and $3)$ **2** Rewrite the b term $(-5x)$ using these two factors. **3** Factorise the first two terms and the last two terms. **4** $(x - 4)$ is a factor of both terms. **5** When two values multiply to make zero, at least one of the values must be zero. **6** Solve these two equations.

Practice

1 Solve

a $6x^2 + 4x = 0$ **b** $28x^2 - 21x = 0$

c $x^2 + 7x + 10 = 0$ **d** $x^2 - 5x + 6 = 0$

e $x^2 - 3x - 4 = 0$ **f** $x^2 + 3x - 10 = 0$

g $x^2 - 10x + 24 = 0$ **h** $x^2 - 36 = 0$

i $x^2 + 3x - 28 = 0$ **j** $x^2 - 6x + 9 = 0$

k $2x^2 - 7x - 4 = 0$ **l** $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$ **b** $x^2 - 3 = 2x$

Hint
Get all terms onto one side of the equation.

c	$x^2 + 5x = 24$	**d**	$x^2 - 42 = x$
e	$x(x + 2) = 2x + 25$	**f**	$x^2 - 30 = 3x - 2$
g	$x(3x + 1) = x^2 + 15$	**h**	$3x(x - 1) = 2(x + 1)$

Solving quadratic equations by completing the square

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r = 0$.

Examples

Example 5 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$x^2 + 6x + 4 = 0$	**1** Write $x^2 + bx + c = 0$ in the form $\left(x+\frac{b}{2}\right)^2-\left(\frac{b}{2}\right)^2+c=0$
$(x + 3)^2 - 9 + 4 = 0$	**2** Simplify.
$(x + 3)^2 - 5 = 0$	**3** Rearrange the equation to work out x. First, add 5 to both sides.
$(x + 3)^2 = 5$	**4** Square root both sides. Remember that the square root of a value gives two answers.
$x + 3 = \pm\sqrt{5}$	**5** Subtract 3 from both sides to solve the equation.
$x = \pm\sqrt{5}-3$	**6** Write down both solutions.
So $x = -\sqrt{5}-3$ or $x = \sqrt{5}-3$	

Example 6 Solve $2x^2 - 7x + 4 = 0$. Give your solutions in surd form.

$2x^2 - 7x + 4 = 0$	**1** Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$
$2\left(x^2 - \frac{7}{2}x\right) + 4 = 0$	
$2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right] + 4 = 0$	**2** Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$
	3 Expand the square brackets.
$2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8} + 4 = 0$	**4** Simplify.
	(continued on next page)
$2\left(x - \frac{7}{4}\right)^2 - \frac{17}{8} = 0$	**5** Rearrange the equation to work out x. First, add $\frac{17}{8}$ to both sides.
$2\left(x - \frac{7}{4}\right)^2 = \frac{17}{8}$	**6** Divide both sides by 2.
$\left(x - \frac{7}{4}\right)^2 = \frac{17}{16}$	**7** Square root both sides. Remember that the square root of a value gives two answers.
$x - \frac{7}{4} = \pm\frac{\sqrt{17}}{4}$	**8** Add $\frac{7}{4}$ to both sides.
$x = \pm\frac{\sqrt{17}}{4} + \frac{7}{4}$	**9** Write down both the solutions.
So $x = \frac{7}{4} - \frac{\sqrt{17}}{4}$ or $x = \frac{7}{4} + \frac{\sqrt{17}}{4}$	

Practice

3 Solve by completing the square.

a $x^2 - 4x - 3 = 0$ **b** $x^2 - 10x + 4 = 0$

c $x^2 + 8x - 5 = 0$ **d** $x^2 - 2x - 6 = 0$

e $2x^2 + 8x - 5 = 0$ **f** $5x^2 + 3x - 4 = 0$

4 Solve by completing the square.

a $(x - 4)(x + 2) = 5$

b $2x^2 + 6x - 7 = 0$

c $x^2 - 5x + 3 = 0$

Hint
Get all terms onto one side of the equation.

Solving quadratic equations by using the formula

A LEVEL LINKS
Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Any quadratic equation of the form $ax^2 + bx + c = 0$ can be solved using the formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- If $b^2 - 4ac$ is negative then the quadratic equation does not have any real solutions.
- It is useful to write down the formula before substituting the values for a, b and c.

Examples

Example 7 Solve $x^2 + 6x + 4 = 0$. Give your solutions in surd form.

$a = 1$, $b = 6$, $c = 4$ $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \dfrac{-6 \pm \sqrt{6^2 - 4(1)(4)}}{2(1)}$	**2** Substitute $a = 1$, $b = 6$, $c = 4$ into the formula.
$x = \dfrac{-6 \pm \sqrt{20}}{2}$	**3** Simplify. The denominator is 2, but this is only because $a = 1$. The denominator will not always be 2.
$x = \dfrac{-6 \pm 2\sqrt{5}}{2}$	**4** Simplify $\sqrt{20}$. $\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$
$x = -3 \pm \sqrt{5}$	**5** Simplify by dividing numerator and denominator by 2.
So $x = -3 - \sqrt{5}$ or $x = \sqrt{5} - 3$	**6** Write down both the solutions.

Example 8 Solve $3x^2 - 7x - 2 = 0$. Give your solutions in surd form.

$a = 3$, $b = -7$, $c = -2$ $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$	**1** Identify a, b and c, making sure you get the signs right and write down the formula. Remember that $-b \pm \sqrt{b^2 - 4ac}$ is all over $2a$, not just part of it.
$x = \frac{-(-7) \pm \sqrt{(-7)^2 - 4(3)(-2)}}{2(3)}$	**2** Substitute $a = 3$, $b = -7$, $c = -2$ into the formula.
$x = \frac{7 \pm \sqrt{73}}{6}$	**3** Simplify. The denominator is 6 when $a = 3$. A common mistake is to always write a denominator of 2.
So $x = \frac{7 - \sqrt{73}}{6}$ or $x = \frac{7 + \sqrt{73}}{6}$	**4** Write down both the solutions.

Practice

5 Solve, giving your solutions in surd form.

a $3x^2 + 6x + 2 = 0$ **b** $2x^2 - 4x - 7 = 0$

6 Solve the equation $x^2 - 7x + 2 = 0$

Give your solutions in the form $\frac{a \pm \sqrt{b}}{c}$, where a, b and c are integers.

7 Solve $10x^2 + 3x + 3 = 5$

Give your solution in surd form.

> **Hint**
> Get all terms onto one side of the equation.

Extend

8 Choose an appropriate method to solve each quadratic equation, giving your answer in surd form when necessary.

a $4x(x - 1) = 3x - 2$

b $10 = (x + 1)^2$

c $x(3x - 1) = 10$

Answers

1 **a** $x = 0$ or $x = -\frac{2}{3}$ **b** $x = 0$ or $x = \frac{3}{4}$

c $x = -5$ or $x = -2$ **d** $x = 2$ or $x = 3$

e $x = -1$ or $x = 4$ **f** $x = -5$ or $x = 2$

g $x = 4$ or $x = 6$ **h** $x = -6$ or $x = 6$

i $x = -7$ or $x = 4$ **j** $x = 3$

k $x = -\frac{1}{2}$ or $x = 4$ **l** $x = -\frac{2}{3}$ or $x = 5$

2 **a** $x = -2$ or $x = 5$ **b** $x = -1$ or $x = 3$

c $x = -8$ or $x = 3$ **d** $x = -6$ or $x = 7$

e $x = -5$ or $x = 5$ **f** $x = -4$ or $x = 7$

g $x = -3$ or $x = 2\frac{1}{2}$ **h** $x = -\frac{1}{3}$ or $x = 2$

3 **a** $x = 2 + \sqrt{7}$ or $x = 2 - \sqrt{7}$ **b** $x = 5 + \sqrt{21}$ or $x = 5 - \sqrt{21}$

c $x = -4 + \sqrt{21}$ or $x = -4 - \sqrt{21}$ **d** $x = 1 + \sqrt{7}$ or $x = 1 - \sqrt{7}$

e $x = -2 + \sqrt{6.5}$ or $x = -2 - \sqrt{6.5}$ **f** $x = \frac{-3+\sqrt{89}}{10}$ or $x = \frac{-3-\sqrt{89}}{10}$

4 **a** $x = 1 + \sqrt{14}$ or $x = 1 - \sqrt{14}$ **b** $x = \frac{-3+\sqrt{23}}{2}$ or $x = \frac{-3-\sqrt{23}}{2}$

c $x = \frac{5+\sqrt{13}}{2}$ or $x = \frac{5-\sqrt{13}}{2}$

5 **a** $x = -1 + \frac{\sqrt{3}}{3}$ or $x = -1 - \frac{\sqrt{3}}{3}$ **b** $x = 1 + \frac{3\sqrt{2}}{2}$ or $x = 1 - \frac{3\sqrt{2}}{2}$

6 $x = \frac{7+\sqrt{41}}{2}$ or $x = \frac{7-\sqrt{41}}{2}$

7 $x = \frac{-3+\sqrt{89}}{20}$ or $x = \frac{-3-\sqrt{89}}{20}$

8 **a** $x = \frac{7+\sqrt{17}}{8}$ or $x = \frac{7-\sqrt{17}}{8}$

b $x = -1 + \sqrt{10}$ or $x = -1 - \sqrt{10}$

c $x = -1\frac{2}{3}$ or $x = 2$

Solving linear simultaneous equations using the elimination method

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Examples

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$3x + y = 5$ $-\quad x + y = 1$ $2x \quad = 4$ So $x = 2$	**1** Subtract the second equation from the first equation to eliminate the y term.
Using $x + y = 1$ $2 + y = 1$ So $y = -1$	**2** To find the value of y, substitute $x = 2$ into one of the original equations.
Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$\begin{array}{r} x + 2y = 13 \\ +\ \ 5x - 2y = 5 \\ \hline 6x \qquad = 18 \end{array}$ So $x = 3$	**1** Add the two equations together to eliminate the y term.
Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$	**2** To find the value of y, substitute $x = 3$ into one of the original equations.
Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Example 3 Solve $2x + 3y = 2$ and $5x + 4y = 12$ simultaneously.

$(2x + 3y = 2) \times 4 \rightarrow \quad 8x + 12y = 8$ $(5x + 4y = 12) \times 3 \rightarrow \quad 15x + 12y = 36$ $7x = 28$ So $x = 4$	**1** Multiply the first equation by 4 and the second equation by 3 to make the coefficient of y the same for both equations. Then subtract the first equation from the second equation to eliminate the y term.
Using $2x + 3y = 2$ $2 \times 4 + 3y = 2$ So $y = -2$	**2** To find the value of y, substitute $x = 4$ into one of the original equations.
Check: equation 1: $2 \times 4 + 3 \times (-2) = 2$ YES equation 2: $5 \times 4 + 4 \times (-2) = 12$ YES	**3** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

1 $4x + y = 8$
$x + y = 5$

2 $3x + y = 7$
$3x + 2y = 5$

3 $4x + y = 3$
$3x - y = 11$

4 $3x + 4y = 7$
$x - 4y = 5$

5 $2x + y = 11$
$x - 3y = 9$

6 $2x + 3y = 11$
$3x + 2y = 4$

Solving linear simultaneous equations using the substitution method

A LEVEL LINKS
Scheme of work: 1c. Equations – quadratic/linear simultaneous
Textbook: Pure Year 1, 3.1 Linear simultaneous equations

Key points

- The subsitution method is the method most commonly used for A level. This is because it is the method used to solve linear and quadratic simultaneous equations.

Examples

Example 4 Solve the simultaneous equations $y = 2x + 1$ and $5x + 3y = 14$

$5x + 3(2x + 1) = 14$	**1** Substitute $2x + 1$ for y into the second equation.
$5x + 6x + 3 = 14$ $11x + 3 = 14$	**2** Expand the brackets and simplify.
$11x = 11$ So $x = 1$	**3** Work out the value of x.
Using $y = 2x + 1$ $y = 2 \times 1 + 1$ So $y = 3$	**4** To find the value of y, substitute $x = 1$ into one of the original equations.
Check: equation 1: $3 = 2 \times 1 + 1$ YES equation 2: $5 \times 1 + 3 \times 3 = 14$ YES	**5** Substitute the values of x and y into both equations to check your answers.

Example 5 Solve $2x - y = 16$ and $4x + 3y = -3$ simultaneously.

$y = 2x - 16$ $4x + 3(2x - 16) = -3$	**1** Rearrange the first equation. **2** Substitute $2x - 16$ for y into the second equation. **3** Expand the brackets and simplify.
$4x + 6x - 48 = -3$ $10x - 48 = -3$ $10x = 45$ So $x = 4\frac{1}{2}$	**4** Work out the value of x.
Using $y = 2x - 16$ $y = 2 \times 4\frac{1}{2} - 16$ So $y = -7$	**5** To find the value of y, substitute $x = 4\frac{1}{2}$ into one of the original equations.
Check: equation 1: $2 \times 4\frac{1}{2} - (-7) = 16$ YES equation 2: $4 \times 4\frac{1}{2} + 3 \times (-7) = -3$ YES	**6** Substitute the values of x and y into both equations to check your answers.

Practice

Solve these simultaneous equations.

7 $y = x - 4$
$2x + 5y = 43$

8 $y = 2x - 3$
$5x - 3y = 11$

9 $2y = 4x + 5$
$9x + 5y = 22$

10 $2x = y - 2$
$8x - 5y = -11$

11 $3x + 4y = 8$
$2x - y = -13$

12 $3y = 4x - 7$
$2y = 3x - 4$

13 $3x = y - 1$
$2y - 2x = 3$

14 $3x + 2y + 1 = 0$
$4y = 8 - x$

Extend

15 Solve the simultaneous equations $3x + 5y - 20 = 0$ and $2(x+y) = \dfrac{3(y-x)}{4}$.

Answers

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$

6 $x = -2, y = 5$

7 $x = 9, y = 5$

8 $x = -2, y = -7$

9 $x = \frac{1}{2}, y = 3\frac{1}{2}$

10 $x = \frac{1}{2}, y = 3$

11 $x = -4, y = 5$

12 $x = -2, y = -5$

13 $x = \frac{1}{4}, y = 1\frac{3}{4}$

14 $x = -2, y = 2\frac{1}{2}$

15 $x = -2\frac{1}{2}, y = 5\frac{1}{2}$

Linear inequalities

A LEVEL LINKS
Scheme of work: 1d. Inequalities – linear and quadratic (including graphical solutions)

Key points

- Solving linear inequalities uses similar methods to those for solving linear equations.
- When you multiply or divide an inequality by a negative number you need to reverse the inequality sign, e.g. < becomes >.

Examples

Example 1 Solve $-8 \leq 4x < 16$

$-8 \leq 4x < 16$ $-2 \leq x < 4$	Divide all three terms by 4.

Example 2 Solve $4 \leq 5x < 10$

$4 \leq 5x < 10$ $\frac{4}{5} \leq x < 2$	Divide all three terms by 5.

Example 3 Solve $2x - 5 < 7$

$2x - 5 < 7$ $2x < 12$ $x < 6$	**1** Add 5 to both sides. **2** Divide both sides by 2.

Example 4 Solve $2 - 5x \geq -8$

$2 - 5x \geq -8$ $-5x \geq -10$ $x \leq 2$	**1** Subtract 2 from both sides. **2** Divide both sides by −5. Remember to reverse the inequality when dividing by a negative number.

Example 5 Solve $4(x - 2) > 3(9 - x)$

$4(x - 2) > 3(9 - x)$ $4x - 8 > 27 - 3x$ $7x - 8 > 27$ $7x > 35$	**1** Expand the brackets. **2** Add $3x$ to both sides. **3** Add 8 to both sides. **4** Divide both sides by 7.

$x > 5$	

Practice

1 Solve these inequalities.

a $4x > 16$ **b** $5x - 7 \leq 3$ **c** $1 \geq 3x + 4$

d $5 - 2x < 12$ **e** $\frac{x}{2} \geq 5$ **f** $8 < 3 - \frac{x}{3}$

2 Solve these inequalities.

a $\frac{x}{5} < -4$ **b** $10 \geq 2x + 3$ **c** $7 - 3x > -5$

3 Solve

a $2 - 4x \geq 18$ **b** $3 \leq 7x + 10 < 45$ **c** $6 - 2x \geq 4$

d $4x + 17 < 2 - x$ **e** $4 - 5x < -3x$ **f** $-4x \geq 24$

4 Solve these inequalities.

a $3t + 1 < t + 6$ **b** $2(3n - 1) \geq n + 5$

5 Solve.

a $3(2 - x) > 2(4 - x) + 4$ **b** $5(4 - x) > 3(5 - x) + 2$

Extend

6 Find the set of values of x for which $2x + 1 > 11$ and $4x - 2 > 16 - 2x$.

Answers

1 **a** $x > 4$ **b** $x \leq 2$ **c** $x \leq -1$

d $x > -\frac{7}{2}$ **e** $x \geq 10$ **f** $x < -15$

2 **a** $x < -20$ **b** $x \leq 3.5$ **c** $x < 4$

3 **a** $x \leq -4$ **b** $-1 \leq x < 5$ **c** $x \leq 1$

d $x < -3$ **e** $x > 2$ **f** $x \leq -6$

4 **a** $t < \frac{5}{2}$ **b** $n \geq \frac{7}{5}$

5 **a** $x < -6$ **b** $x < \frac{3}{2}$

6 $x > 5$ (which also satisfies $x > 3$)

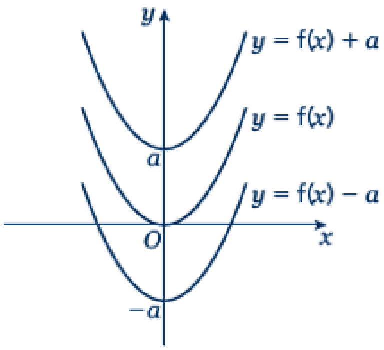
Translating graphs

A LEVEL LINKS
Scheme of work: 1f. Transformations – transforming graphs – f(x) notation

Key points

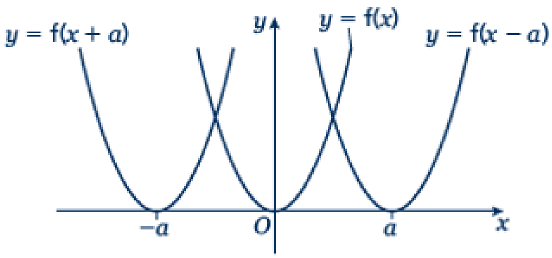
- The transformation $y = \mathrm{f}(x) \pm a$ is a translation of $y = \mathrm{f}(x)$ parallel to the y-axis; it is a vertical translation.

  As shown on the graph,
  - $y = \mathrm{f}(x) + a$ translates $y = \mathrm{f}(x)$ up
  - $y = \mathrm{f}(x) - a$ translates $y = \mathrm{f}(x)$ down.



- The transformation $y = \mathrm{f}(x \pm a)$ is a translation of $y = \mathrm{f}(x)$ parallel to the x-axis; it is a horizontal translation.

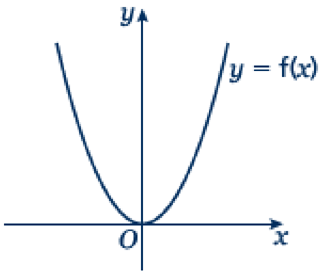
  As shown on the graph,
  - $y = \mathrm{f}(x + a)$ translates $y = \mathrm{f}(x)$ to the left
  - $y = \mathrm{f}(x - a)$ translates $y = \mathrm{f}(x)$ to the right.

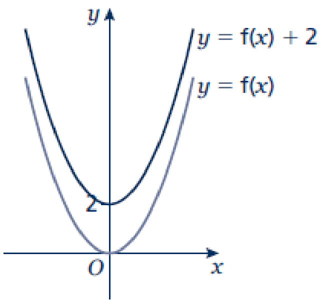


Examples

Example 1 The graph shows the function $y = \mathrm{f}(x)$.

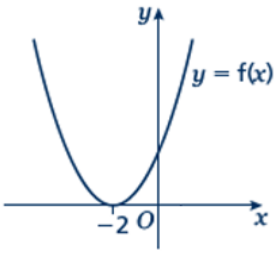
Sketch the graph of $y = \mathrm{f}(x) + 2$.

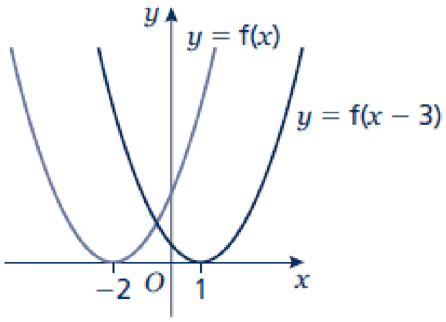


	For the function $y = \mathrm{f}(x) + 2$ translate the function $y = \mathrm{f}(x)$ 2 units up.

Example 2 The graph shows the function $y = f(x)$.

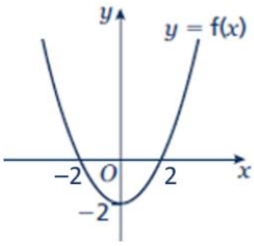
Sketch the graph of $y = f(x - 3)$.



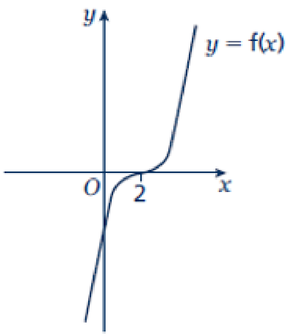
	For the function $y = f(x - 3)$ translate the function $y = f(x)$ 3 units right.

Practice

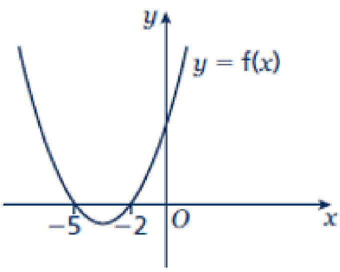
1 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = f(x) + 4$ and $y = f(x + 2)$.



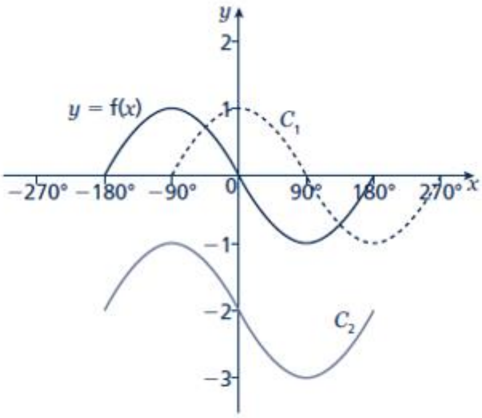
2 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch and label the graphs of $y = f(x + 3)$ and $y = f(x) – 3$.



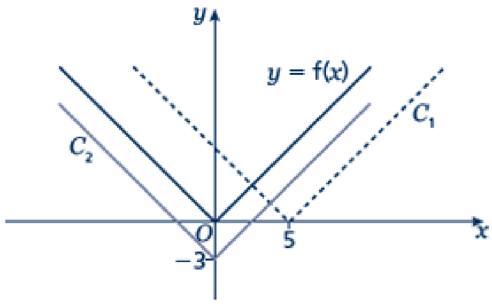
3 The graph shows the function $y = f(x)$.
Copy the graph and on the same axes sketch the graph of $y = f(x – 5)$.



4 The graph shows the function $y = \mathrm{f}(x)$ and two transformations of $y = \mathrm{f}(x)$, labelled C_1 and C_2. Write down the equations of the translated curves C and C_2 in function form.



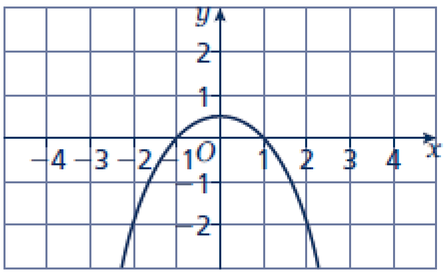
5 The graph shows the function $y = \mathrm{f}(x)$ and two transformations of $y = \mathrm{f}(x)$, labelled C_1 and C_2. Write down the equations of the translated curves C_1 and C_2 in function form.



6 The graph shows the function $y = \mathrm{f}(x)$.

a Sketch the graph of $y = \mathrm{f}(x) + 2$

b Sketch the graph of $y = \mathrm{f}(x + 2)$

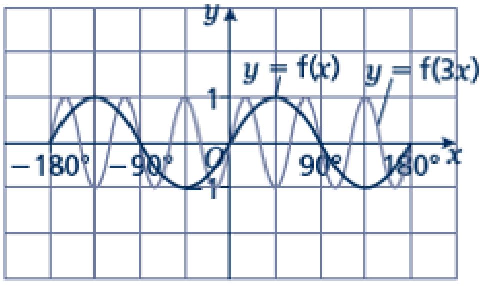


Stretching graphs

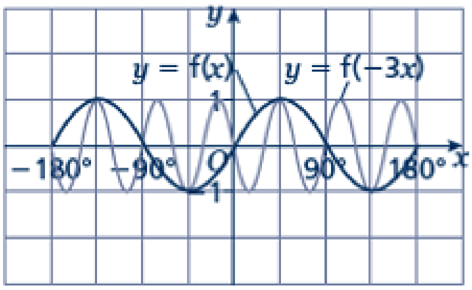
A LEVEL LINKS
Scheme of work: 1f. Transformations – transforming graphs – f(x) notation
Textbook: Pure Year 1, 4.6 Stretching graphs

Key points

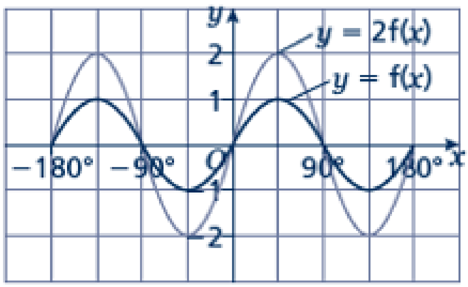
- The transformation $y = f(ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ parallel the x-axis.



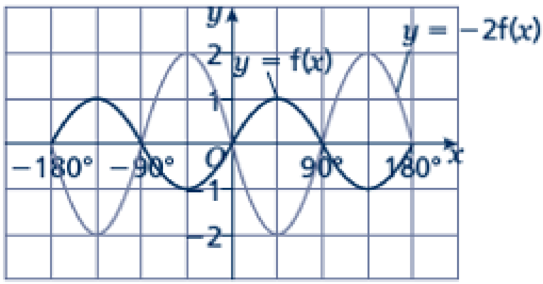
- The transformation $y = f(–ax)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{a}$ paralle the x-axis and then a reflection in the y-axis.



- The transformation $y = af(x)$ is a vertical stret of $y = f(x)$ with scale factor a parallel to the y-axis.



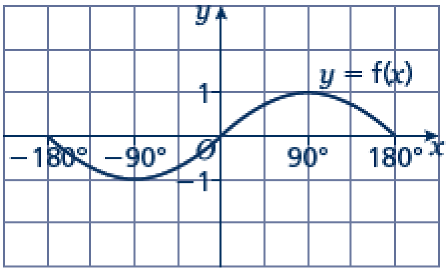
- The transformation $y = –af(x)$ is a vertical stretch of $y = f(x)$ with scale factor a parallel the y-axis and then a reflection in the x-axis.

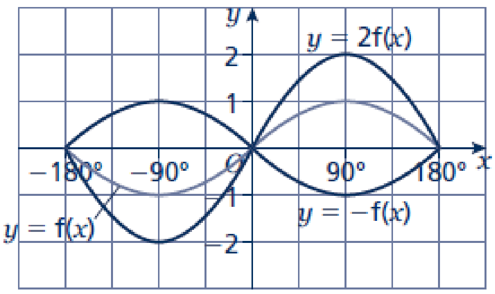


Examples

Example 3 The graph shows the function $y = f(x)$.

Sketch and label the graphs of $y = 2f(x)$ and $y = -f(x)$.



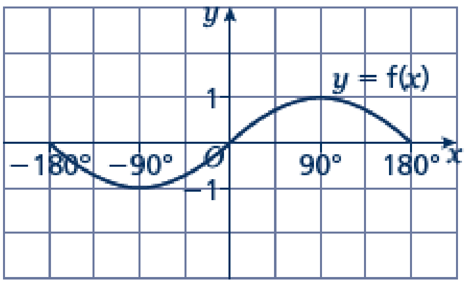


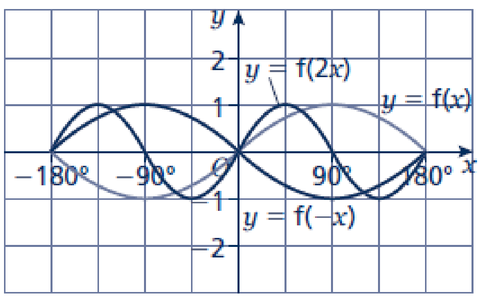
The function $y = 2f(x)$ is a vertical stretch of $y = f(x)$ with scale factor 2 parallel to the y-axis.

The function $y = -f(x)$ is a reflection of $y = f(x)$ in the x-axis.

Example 4 The graph shows the function $y = f(x)$.

Sketch and label the graphs of $y = f(2x)$ and $y = f(-x)$.





The function $y = f(2x)$ is a horizontal stretch of $y = f(x)$ with scale factor $\frac{1}{2}$ parallel to the x-axis.

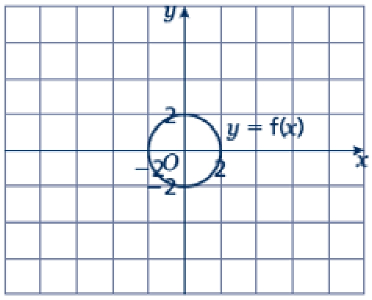
The function $y = f(-x)$ is a reflection of $y = f(x)$ in the y-axis.

Practice

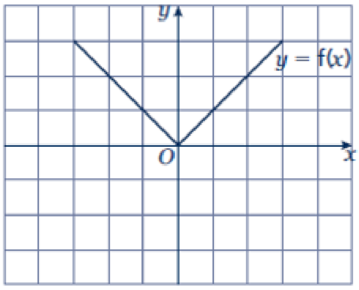
7 The graph shows the function $y = \mathrm{f}(x)$.

a Copy the graph and on the same axes sketch and label the graph of $y = 3\mathrm{f}(x)$.

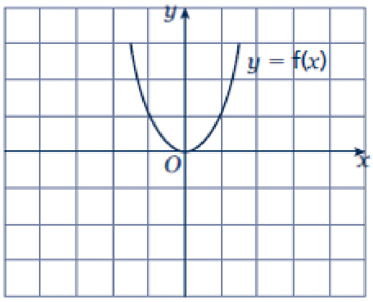
b Make another copy of the graph and on the same axes sketch and label the graph of $y = \mathrm{f}(2x)$.



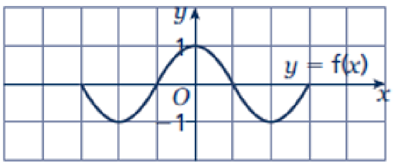
8 The graph shows the function $y = \mathrm{f}(x)$.
Copy the graph and on the same axes sketch and label the graphs of
$y = -2\mathrm{f}(x)$ and $y = \mathrm{f}(3x)$.



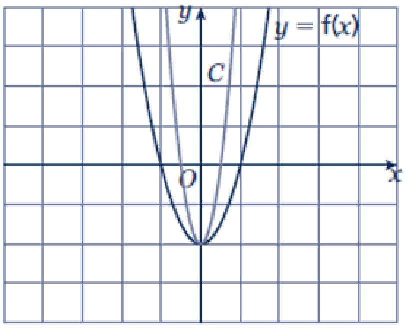
9 The graph shows the function $y = \mathrm{f}(x)$.
Copy the graph and, on the same axes, sketch and label the graphs of
$y = -\mathrm{f}(x)$ and $y = \mathrm{f}\left(\frac{1}{2}x\right)$.



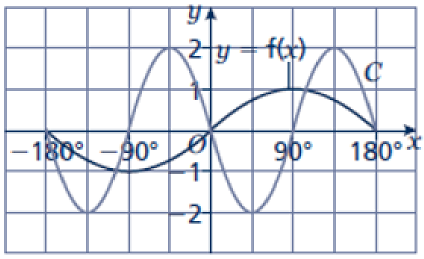
10 The graph shows the function $y = \mathrm{f}(x)$.
Copy the graph and, on the same axes, sketch the graph of $y = -\mathrm{f}(2x)$.



11 The graph shows the function $y = \mathrm{f}(x)$ and a transformation, labelled C.
Write down the equation of the translated curve C in function form.



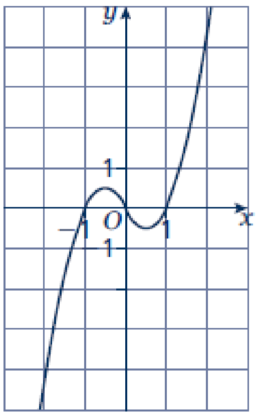
12 The graph shows the function $y = f(x)$ and a transformation labelled C.
Write down the equation of the translated curve C in function form.



13 The graph shows the function $y = f(x)$.

a Sketch the graph of $y = -f(x)$.

b Sketch the graph of $y = 2f(x)$.



Extend

14 **a** Sketch and label the graph of $y = f(x)$, where $f(x) = (x - 1)(x + 1)$.

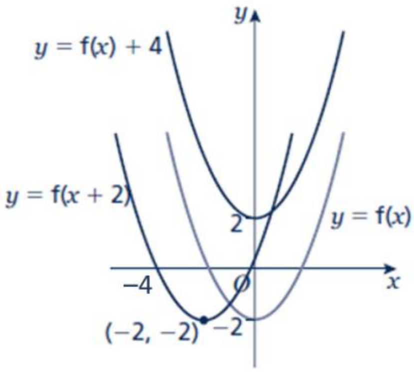
b On the same axes, sketch and label the graphs of $y = f(x) - 2$ and $y = f(x + 2)$.

15 **a** Sketch and label the graph of $y = f(x)$, where $f(x) = -(x + 1)(x - 2)$.

b On the same axes, sketch and label the graph of $y = f\left(-\frac{1}{2}x\right)$.

Answers

1



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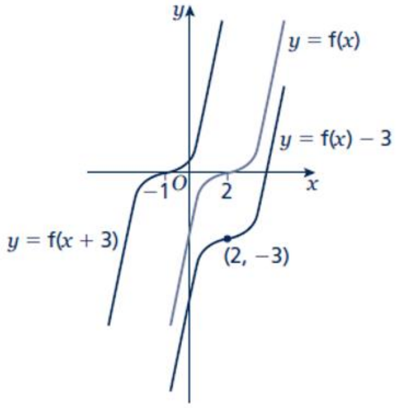

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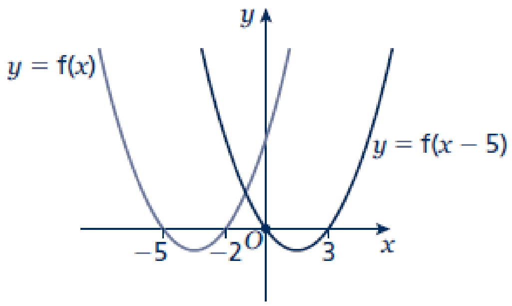
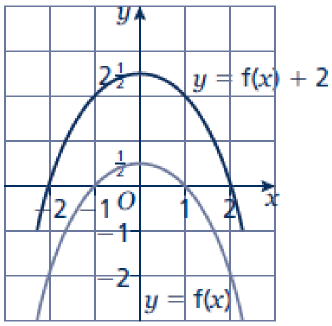


4 C_1: $y = f(x - 90°)$
C_2: $y = f(x) - 2$

5 C_1: $y = f(x - 5)$
C_2: $y = f(x) - 3$

6 **a**



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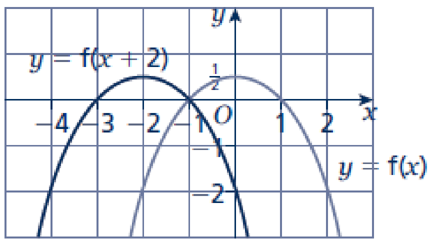
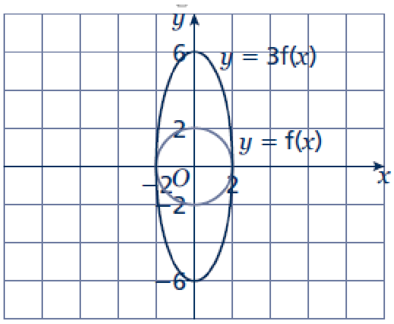


7 **a**



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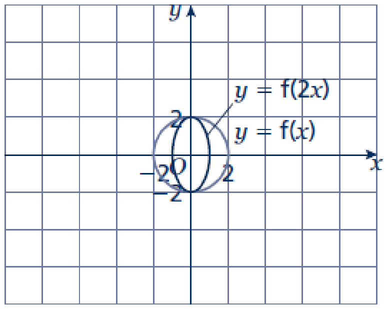

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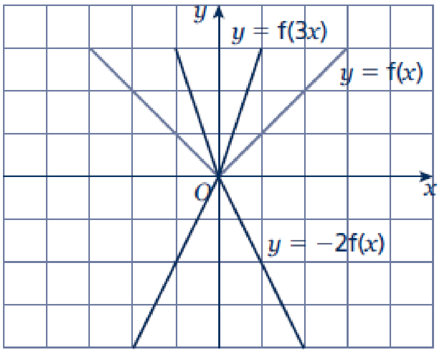

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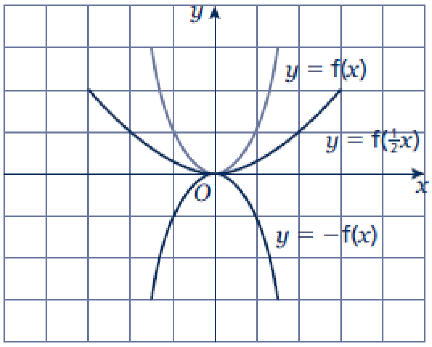

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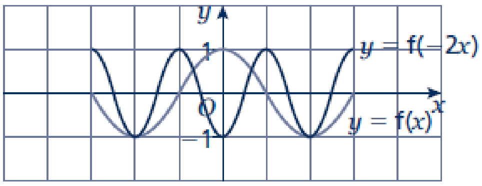
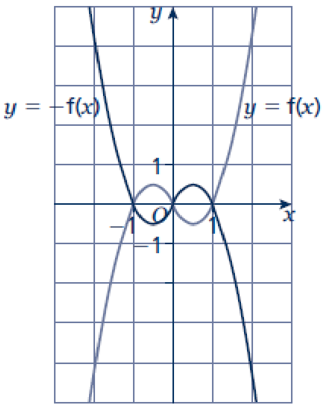


11 $y = \mathrm{f}(2x)$

12 $y = -2\mathrm{f}(2x)$ or $y = 2\mathrm{f}(-2x)$

13 **a**



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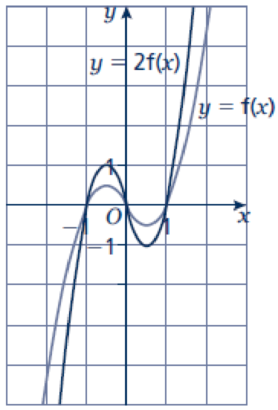

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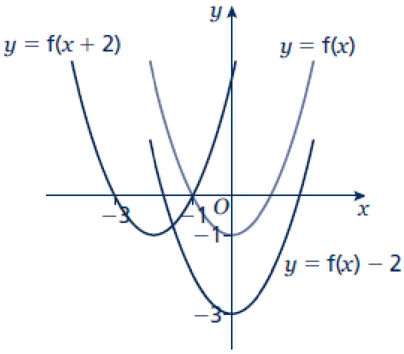

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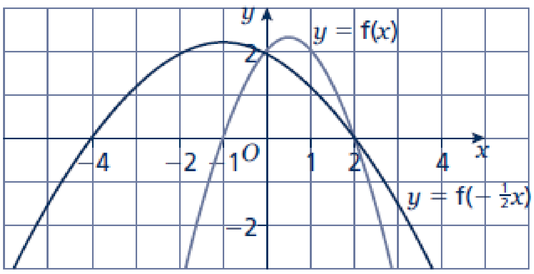
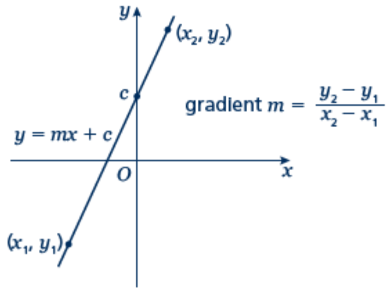


Straight line graphs

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- A straight line has the equation $y = mx + c$, where m is the gradient and c is the y-intercept (where $x = 0$).
- The equation of a straight line can be written in the form $ax + by + c = 0$, where a, b and c are integers.
- When given the coordinates (x_1, y_1) and (x_2, y_2) of two points on a line the gradient is calculated using the formula $m = \frac{y_2 - y_1}{x_2 - x_1}$



Examples

Example 1 A straight line has gradient $-\frac{1}{2}$ and y-intercept 3.

Write the equation of the line in the form $ax + by + c = 0$.

$m = -\frac{1}{2}$ and $c = 3$ So $y = -\frac{1}{2}x + 3$ $\frac{1}{2}x + y - 3 = 0$ $x + 2y - 6 = 0$	**1** A straight line has equation $y = mx + c$. Substitute the gradient and y-intercept given in the question into this equation. **2** Rearrange the equation so all the terms are on one side and 0 is on the other side. **3** Multiply both sides by 2 to eliminate the denominator.

Example 2 Find the gradient and the y-intercept of the line with the equation $3y - 2x + 4 = 0$.

$3y - 2x + 4 = 0$ $3y = 2x - 4$ $y = \frac{2}{3}x - \frac{4}{3}$ Gradient = $m = \frac{2}{3}$ y-intercept = $c = -\frac{4}{3}$	**1** Make y the subject of the equation. **2** Divide all the terms by three to get the equation in the form $y = \ldots$ **3** In the form $y = mx + c$, the gradient is m and the y-intercept is c.

Example 3 Find the equation of the line which passes through the point (5, 13) and has gradient 3.

$m = 3$ $y = 3x + c$	**1** Substitute the gradient given in the question into the equation of a straight line $y = mx + c$.
$13 = 3 \times 5 + c$	**2** Substitute the coordinates $x = 5$ and $y = 13$ into the equation. **3** Simplify and solve the equation.
$13 = 15 + c$ $c = -2$ $y = 3x - 2$	**4** Substitute $c = -2$ into the equation $y = 3x + c$

Example 4 Find the equation of the line passing through the points with coordinates (2, 4) and (8, 7).

$x_1 = 2$, $x_2 = 8$, $y_1 = 4$ and $y_2 = 7$ $m = \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{7-4}{8-2} = \dfrac{3}{6} = \dfrac{1}{2}$	**1** Substitute the coordinates into the equation $m = \dfrac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$y = \dfrac{1}{2}x + c$	**2** Substitute the gradient into the equation of a straight line $y = mx + c$.
$4 = \dfrac{1}{2} \times 2 + c$ $c = 3$	**3** Substitute the coordinates of either point into the equation. **4** Simplify and solve the equation.
$y = \dfrac{1}{2}x + 3$	**5** Substitute $c = 3$ into the equation $y = \dfrac{1}{2}x + c$

Practice

1 Find the gradient and the y-intercept of the following equations.

a $y = 3x + 5$ **b** $y = -\frac{1}{2}x - 7$

c $2y = 4x - 3$ **d** $x + y = 5$

e $2x - 3y - 7 = 0$ **f** $5x + y - 4 = 0$

> **Hint**
> Rearrange the equations to the form $y = mx + c$

2 Copy and complete the table, giving the equation of the line in the form $y = mx + c$.

Gradient	***y*-intercept**	**Equation of the line**
5	0	
–3	2	
4	–7	

3 Find, in the form $ax + by + c = 0$ where a, b and c are integers, an equation for each of the lines with the following gradients and y-intercepts.

a gradient $-\frac{1}{2}$, y-intercept –7

b gradient 2, y-intercept 0

c gradient $\frac{2}{3}$, y-intercept 4

d gradient –1.2, y-intercept –2

4 Write an equation for the line which passes though the point (2, 5) and has gradient 4.

5 Write an equation for the line which passes through the point (6, 3) and has gradient $-\frac{2}{3}$

6 Write an equation for the line passing through each of the following pairs of points.

a (4, 5), (10, 17)

b (0, 6), (–4, 8)

c (–1, –7), (5, 23)

d (3, 10), (4, 7)

Extend

7 The equation of a line is $2y + 3x - 6 = 0$.
Write as much information as possible about this line.

Answers

1 **a** $m = 3, c = 5$ **b** $m = -\frac{1}{2}, c = -7$

c $m = 2, c = -\frac{3}{2}$ **d** $m = -1, c = 5$

e $m = \frac{2}{3}, c = -\frac{7}{3}$ or $-2\frac{1}{3}$ **f** $m = -5, c = 4$

2

Gradient	*y*-intercept	Equation of the line
5	0	$y = 5x$
–3	2	$y = -3x + 2$
4	–7	$y = 4x - 7$

3 **a** $x + 2y + 14 = 0$ **b** $2x - y = 0$

c $2x - 3y + 12 = 0$ **d** $6x + 5y + 10 = 0$

4 $y = 4x - 3$

5 $y = -\frac{2}{3}x + 7$

6 **a** $y = 2x - 3$ **b** $y = -\frac{1}{2}x + 6$

c $y = 5x - 2$ **d** $y = -3x + 19$

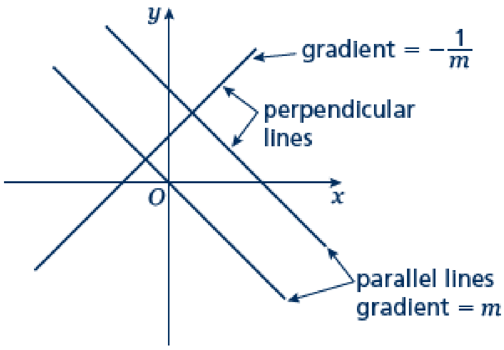
7 $y = -\frac{3}{2}x + 3$, the gradient is $-\frac{3}{2}$ and the *y*-intercept is 3.
The line intercepts the axes at (0, 3) and (2, 0).
Students may sketch the line or give coordinates that lie on the line such as $\left(1, \frac{3}{2}\right)$ or $(4, -3)$.

Parallel and perpendicular lines

A LEVEL LINKS
Scheme of work: 2a. Straight-line graphs, parallel/perpendicular, length and area problems

Key points

- When lines are parallel they have the same gradient.
- A line perpendicular to the line with equation $y = mx + c$ has gradient $-\frac{1}{m}$.



Examples

Example 1 Find the equation of the line parallel to $y = 2x + 4$ which passes through the point (4, 9).

$y = 2x + 4$ $m = 2$ $y = 2x + c$	**1** As the lines are parallel they have the same gradient. **2** Substitute $m = 2$ into the equation of a straight line $y = mx + c$.
$9 = 2 \times 4 + c$	**3** Substitute the coordinates into the equation $y = 2x + c$
$9 = 8 + c$ $c = 1$	**4** Simplify and solve the equation.
$y = 2x + 1$	**5** Substitute $c = 1$ into the equation $y = 2x + c$

Example 2 Find the equation of the line perpendicular to $y = 2x - 3$ which passes through the point (−2, 5).

$y = 2x - 3$ $m = 2$ $-\frac{1}{m} = -\frac{1}{2}$ $y = -\frac{1}{2}x + c$ $5 = -\frac{1}{2} \times (-2) + c$	**1** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$. **2** Substitute $m = -\frac{1}{2}$ into $y = mx + c$. **3** Substitute the coordinates (–2, 5) into the equation $y = -\frac{1}{2}x + c$ **4** Simplify and solve the equation.
$5 = 1 + c$ $c = 4$ $y = -\frac{1}{2}x + 4$	**5** Substitute $c = 4$ into $y = -\frac{1}{2}x + c$.

Example 3 A line passes through the points (0, 5) and (9, −1).
Find the equation of the line which is perpendicular to the line and passes through its midpoint.

$x_1 = 0$, $x_2 = 9$, $y_1 = 5$ and $y_2 = -1$ $m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1-5}{9-0}$ $= \frac{-6}{9} = -\frac{2}{3}$	**1** Substitute the coordinates into the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ to work out the gradient of the line.
$-\frac{1}{m} = \frac{3}{2}$ $y = \frac{3}{2}x + c$	**2** As the lines are perpendicular, the gradient of the perpendicular line is $-\frac{1}{m}$. **3** Substitute the gradient into the equation $y = mx + c$. **4** Work out the coordinates of the midpoint of the line.
Midpoint $= \left(\frac{0+9}{2}, \frac{5+(-1)}{2}\right) = \left(\frac{9}{2}, 2\right)$ $2 = \frac{3}{2} \times \frac{9}{2} + c$ $c = -\frac{19}{4}$ $y = \frac{3}{2}x - \frac{19}{4}$	**5** Substitute the coordinates of the midpoint into the equation. **6** Simplify and solve the equation. **7** Substitute $c = -\frac{19}{4}$ into the equation $y = \frac{3}{2}x + c$.

Practice

1 Find the equation of the line parallel to each of the given lines and which passes through each of the given points.

a $y = 3x + 1$ (3, 2) **b** $y = 3 - 2x$ (1, 3)

c $2x + 4y + 3 = 0$ (6, –3) **d** $2y - 3x + 2 = 0$ (8, 20)

2 Find the equation of the line perpendicular to $y = \frac{1}{2}x - 3$ which passes through the point (–5, 3).

Hint

If $m = \frac{a}{b}$ then the negative reciprocal $-\frac{1}{m} = -\frac{b}{a}$

3 Find the equation of the line perpendicular to each of the given lines and which passes through each of the given points.

a $y = 2x - 6$ (4, 0) **b** $y = -\frac{1}{3}x + \frac{1}{2}$ (2, 13)

c $x - 4y - 4 = 0$ (5, 15) **d** $5y + 2x - 5 = 0$ (6, 7)

4 In each case find an equation for the line passing through the origin which is also perpendicular to the line joining the two points given.

a (4, 3), (–2, –9) **b** (0, 3), (–10, 8)

Extend

5 Work out whether these pairs of lines are parallel, perpendicular or neither.

a $y = 2x + 3$
$y = 2x - 7$

b $y = 3x$
$2x + y - 3 = 0$

c $y = 4x - 3$
$4y + x = 2$

d $3x - y + 5 = 0$
$x + 3y = 1$

e $2x + 5y - 1 = 0$
$y = 2x + 7$

f $2x - y = 6$
$6x - 3y + 3 = 0$

6 The straight line $\mathbf{L_1}$ passes through the points A and B with coordinates (–4, 4) and (2, 1), respectively.

a Find the equation of $\mathbf{L_1}$ in the form $ax + by + c = 0$

The line $\mathbf{L_2}$ is parallel to the line $\mathbf{L_1}$ and passes through the point C with coordinates (–8, 3).

b Find the equation of $\mathbf{L_2}$ in the form $ax + by + c = 0$

The line $\mathbf{L_3}$ is perpendicular to the line $\mathbf{L_1}$ and passes through the origin.

c Find an equation of $\mathbf{L_3}$

Answers

1 **a** $y = 3x - 7$ **b** $y = -2x + 5$

c $y = -\frac{1}{2}x$ **d** $y = \frac{3}{2}x + 8$

2 $y = -2x - 7$

3 **a** $y = -\frac{1}{2}x + 2$ **b** $y = 3x + 7$

c $y = -4x + 35$ **d** $y = \frac{5}{2}x - 8$

4 **a** $y = -\frac{1}{2}x$ **b** $y = 2x$

5 **a** Parallel **b** Neither **c** Perpendicular

d Perpendicular **e** Neither **f** Parallel

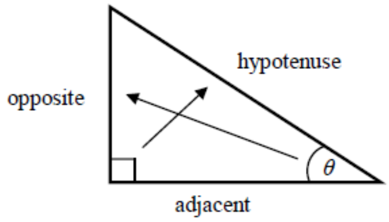
6 **a** $x + 2y - 4 = 0$ **b** $x + 2y + 2 = 0$ **c** $y = 2x$

Trigonometry in right-angled triangles

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs

Key points

- In a right-angled triangle:
  - the side opposite the right angle is called the hypotenuse
  - the side opposite the angle θ is called the opposite
  - the side next to the angle θ is called the adjacent.



- In a right-angled triangle:
  - the ratio of the opposite side to the hypotenuse is the sine of angle θ, $\sin\theta = \frac{\text{opp}}{\text{hyp}}$
  - the ratio of the adjacent side to the hypotenuse is the cosine of angle θ, $\cos\theta = \frac{\text{adj}}{\text{hyp}}$
  - the ratio of the opposite side to the adjacent side is the tangent of angle θ, $\tan\theta = \frac{\text{opp}}{\text{adj}}$

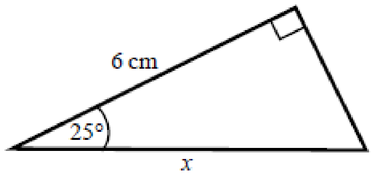
- If the lengths of two sides of a right-angled triangle are given, you can find a missing angle using the inverse trigonometric functions: $\sin^{-1}$, $\cos^{-1}$, $\tan^{-1}$.

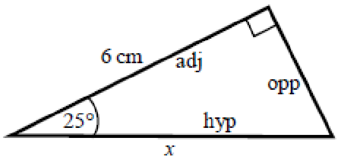
- The sine, cosine and tangent of some angles may be written exactly.

	0	**30°**	**45°**	**60°**	**90°**
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	

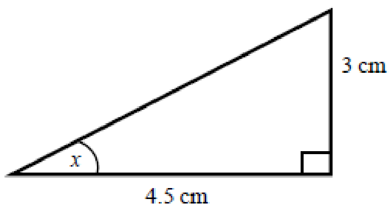
Examples

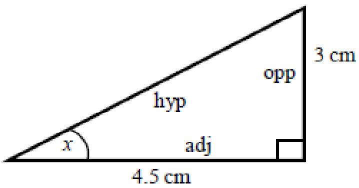
Example 1 Calculate the length of side x.
Give your answer correct to 3 significant figures.



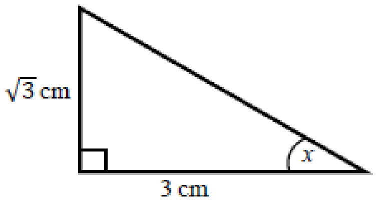
	**1** Always start by labelling the sides.
$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$	**2** You are given the adjacent and the hypotenuse so use the cosine ratio.
$\cos 25° = \dfrac{6}{x}$	**3** Substitute the sides and angle into the cosine ratio.
$x = \dfrac{6}{\cos 25°}$	**4** Rearrange to make x the subject.
$x = 6.620\ 267\ 5...$	**5** Use your calculator to work out $6 \div \cos 25°$.
$x = 6.62$ cm	**6** Round your answer to 3 significant figures and write the units in your answer.

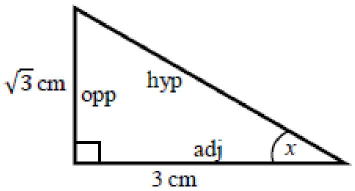
Example 2 Calculate the size of angle x.
Give your answer correct to 3 significant figures.



	**1** Always start by labelling the sides.
$\tan \theta = \frac{\text{opp}}{\text{adj}}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \frac{3}{4.5}$	**3** Substitute the sides and angle into the tangent ratio.
$x = \tan^{-1}\left(\frac{3}{4.5}\right)$	**4** Use $\tan^{-1}$ to find the angle.
$x = 33.690\ 067\ 5...$	**5** Use your calculator to work out $\tan^{-1}(3 \div 4.5)$.
$x = 33.7°$	**6** Round your answer to 3 significant figures and write the units in your answer.

Example 3 Calculate the exact size of angle *x*.

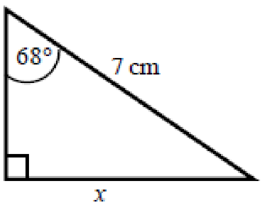


	**1** Always start by labelling the sides.
$\tan \theta = \frac{\text{opp}}{\text{adj}}$	**2** You are given the opposite and the adjacent so use the tangent ratio.
$\tan x = \frac{\sqrt{3}}{3}$	**3** Substitute the sides and angle into the tangent ratio. **4** Use the table from the key points to find the angle.
$x = 30°$	

Practice

1 Calculate the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

a



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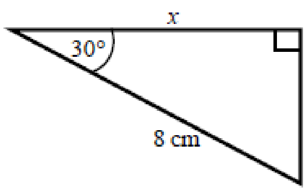

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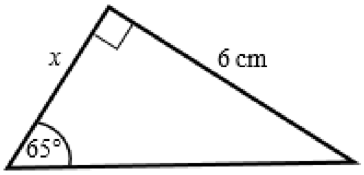

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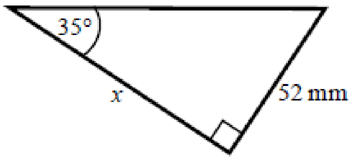

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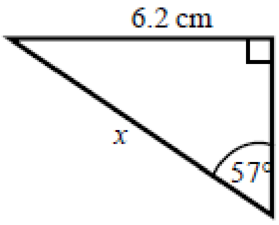

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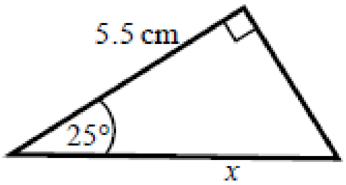
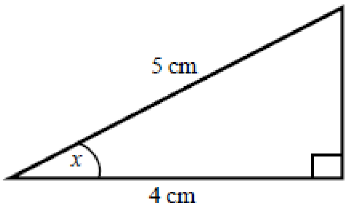


2 Calculate the size of angle x in each triangle.
Give your answers correct to 1 decimal place.

a



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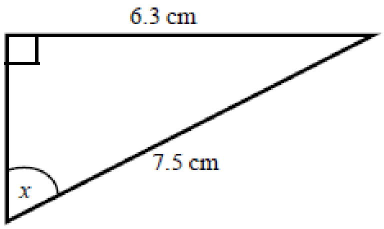

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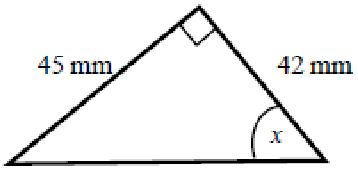
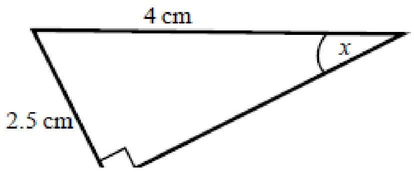
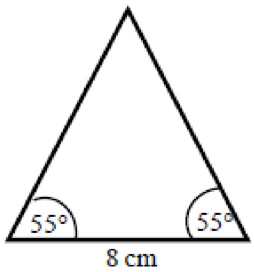


d



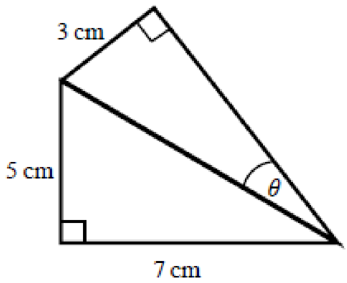
3 Work out the height of the isosceles triangle.
Give your answer correct to 3 significant figures.

Hint:
Split the triangle into two right-angled triangles.



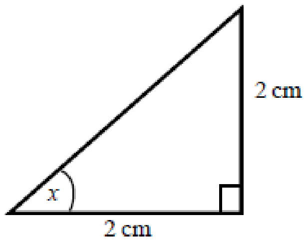
4 Calculate the size of angle θ.
Give your answer correct to 1 decimal place.

Hint:
First work out the length of the common side to both triangles, leaving your answer in surd form.

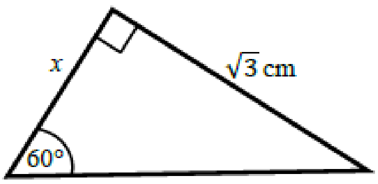


5 Find the exact value of x in each triangle.

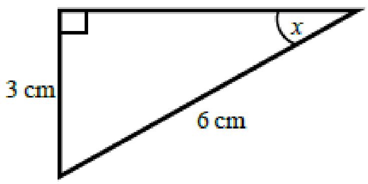
a



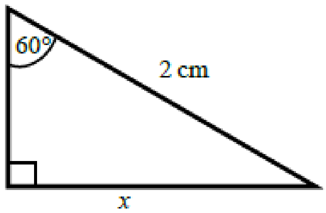
b



c



d

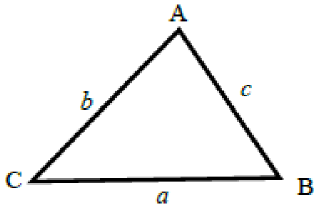


The cosine rule

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.1 The cosine rule

Key points

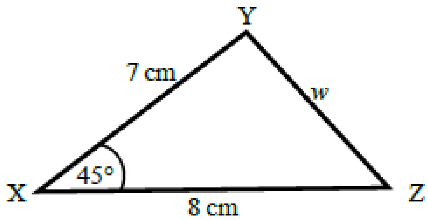
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

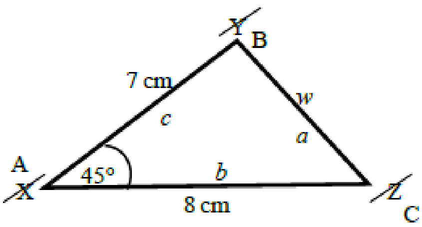


- You can use the cosine rule to find the length of a side when two sides and the included angle are given.
- To calculate an unknown side use the formula $a^2 = b^2 + c^2 - 2bc\cos A$.
- Alternatively, you can use the cosine rule to find an unknown angle if the lengths of all three sides are given.
- To calculate an unknown angle use the formula $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$.

Examples

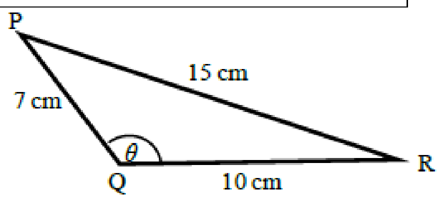
Example 4 Work out the length of side *w*.
Give your answer correct to 3 significant figures.

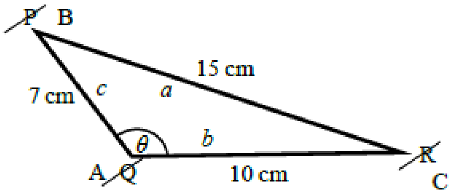


	**1** Always start by labelling the angles and sides.
$a^2 = b^2 + c^2 - 2bc\cos A$	**2** Write the cosine rule to find the side.
$w^2 = 8^2 + 7^2 - 2 \times 8 \times 7 \times \cos 45°$	**3** Substitute the values *a*, *b* and *A* into the formula.
$w^2 = 33.804\,040\,51...$	**4** Use a calculator to find w^2 and then *w*.
$w = \sqrt{33.80404051}$	**5** Round your final answer to 3 significant figures and write the units in your answer.

$w = 5.81$ cm	

Example 5 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

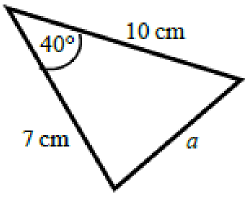


	**1** Always start by labelling the angles and sides.
$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ $\cos\theta = \frac{10^2 + 7^2 - 15^2}{2 \times 10 \times 7}$	**2** Write the cosine rule to find the angle. **3** Substitute the values a, b and c into the formula.
$\cos\theta = \frac{-76}{140}$	**4** Use $\cos^{-1}$ to find the angle.
$\theta = 122.878\ 349...$	**5** Use your calculator to work out $\cos^{-1}(-76 \div 140)$.
$\theta = 122.9°$	**6** Round your answer to 1 decimal place and write the units in your answer.

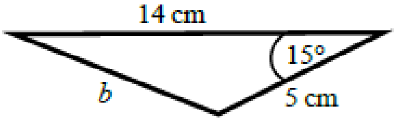
Practice

6 Work out the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

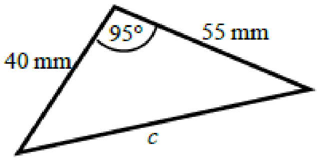
a



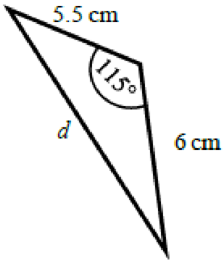
b



c

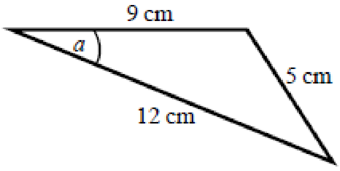


d

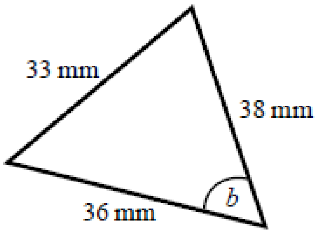


7 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

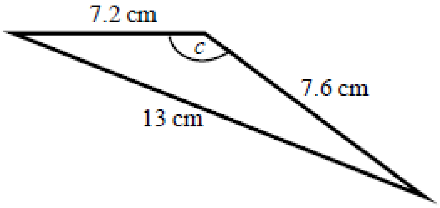
a



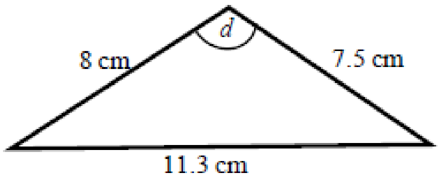
b



c

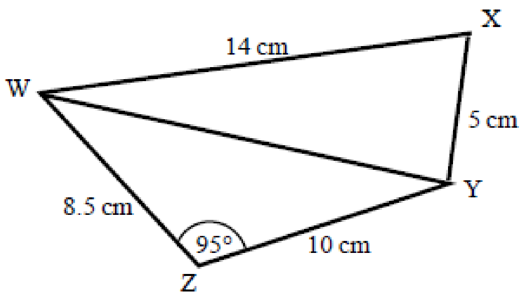


d



8 **a** Work out the length of WY.
Give your answer correct to 3 significant figures.

b Work out the size of angle WXY.
Give your answer correct to 1 decimal place.

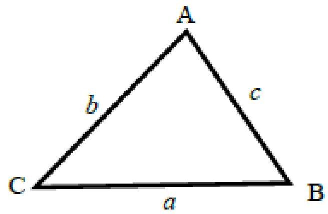


The sine rule

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.2 The sine rule

Key points

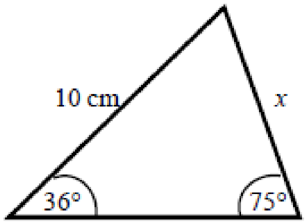
- a is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.

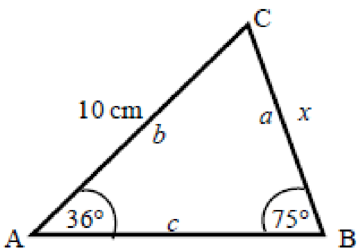


- You can use the sine rule to find the length of a side when its opposite angle and another opposite side and angle are given.
- To calculate an unknown side use the formula $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$.
- Alternatively, you can use the sine rule to find an unknown angle if the opposite side and another opposite side and angle are given.
- To calculate an unknown angle use the formula $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$.

Examples

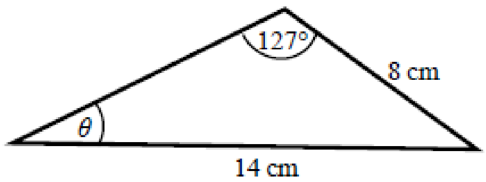
Example 6 Work out the length of side x.
Give your answer correct to 3 significant figures.

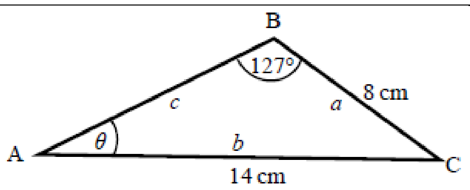


	**1** Always start by labelling the angles and sides.
$\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$	**2** Write the sine rule to find the side.
$\dfrac{x}{\sin 36°} = \dfrac{10}{\sin 75°}$	**3** Substitute the values a, b, A and B into the formula.
$x = \dfrac{10 \times \sin 36°}{\sin 75°}$ $x = 6.09$ cm	**4** Rearrange to make x the subject.

	5 Round your answer to 3 significant figures and write the units in your answer.

Example 7 Work out the size of angle θ.
Give your answer correct to 1 decimal place.

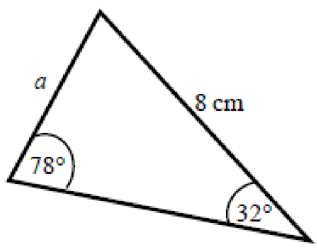


	**1** Always start by labelling the angles and sides.
$\frac{\sin A}{a} = \frac{\sin B}{b}$	**2** Write the sine rule to find the angle.
$\frac{\sin\theta}{8} = \frac{\sin 127°}{14}$	**3** Substitute the values a, b, A and B into the formula.
$\sin\theta = \frac{8 \times \sin 127°}{14}$	**4** Rearrange to make $\sin\theta$ the subject.
$\theta = 27.2°$	**5** Use $\sin^{-1}$ to find the angle. Round your answer to 1 decimal place and write the units in your answer.

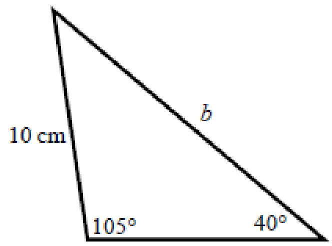
Practice

9 Find the length of the unknown side in each triangle.
Give your answers correct to 3 significant figures.

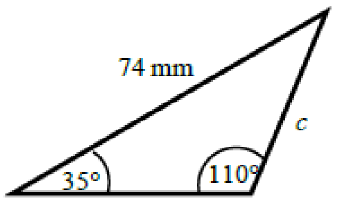
a



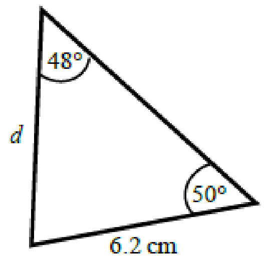
b



c

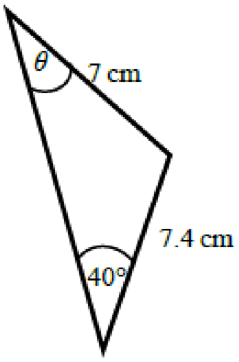


d

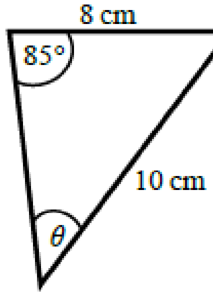


10 Calculate the angles labelled θ in each triangle.
Give your answer correct to 1 decimal place.

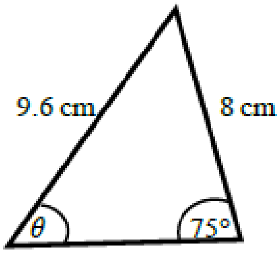
a



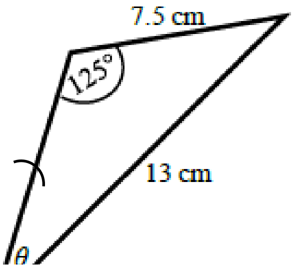
b



c

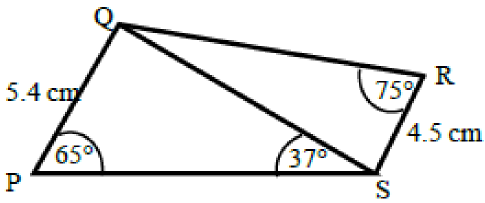


d



11 **a** Work out the length of QS.
Give your answer correct to 3 significant figures.

b Work out the size of angle RQS.
Give your answer correct to 1 decimal place.

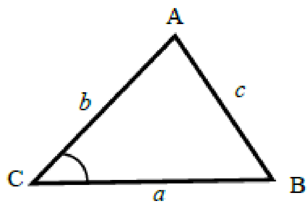


Areas of triangles

A LEVEL LINKS
Scheme of work: 4a. Trigonometric ratios and graphs
Textbook: Pure Year 1, 9.3 Areas of triangles

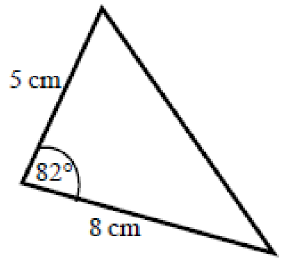
Key points

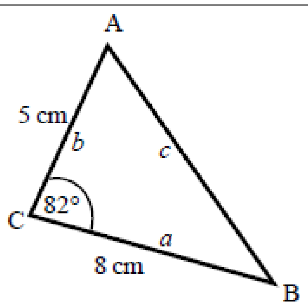
- *a* is the side opposite angle A.
  b is the side opposite angle B.
  c is the side opposite angle C.
- The area of the triangle is $\frac{1}{2}ab\sin C$.



Examples

Example 8 Find the area of the triangle.

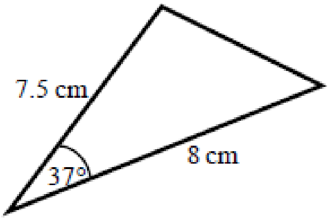


	**1** Always start by labelling the sides and angles of the triangle.
Area = $\frac{1}{2}ab\sin C$	**2** State the formula for the area of a triangle.
Area = $\frac{1}{2}\times 8\times 5\times \sin 82°$	**3** Substitute the values of *a*, *b* and *C* into the formula for the area of a triangle.
Area = 19.805 361...	**4** Use a calculator to find the area.
Area = 19.8 cm^2	**5** Round your answer to 3 significant figures and write the units in your answer.

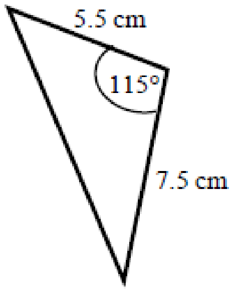
Practice

12 Work out the area of each triangle.
Give your answers correct to 3 significant figures.

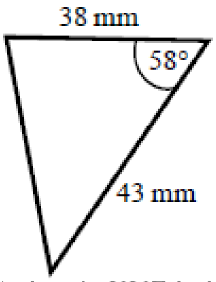
a



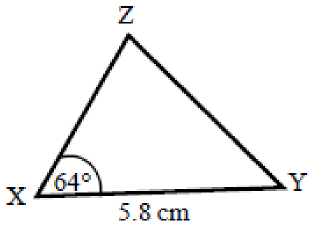
b



c



13 The area of triangle XYZ is 13.3 cm^2.
Work out the length of XZ.



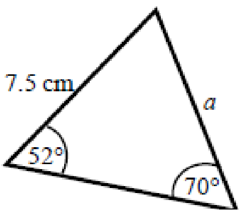
Hint:
Rearrange the formula to make a side the subject.

Extend

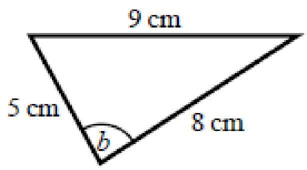
14 Find the size of each lettered angle or side.
Give your answers correct to 3 significant figures.

Hint:
For each one, decide whether to use the cosine or sine rule.

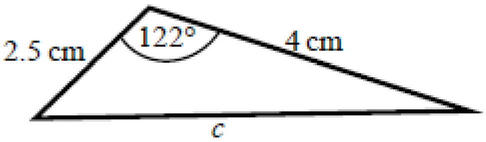
a



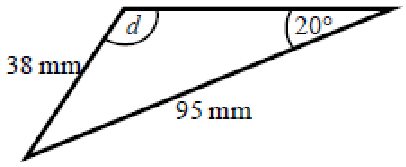
b



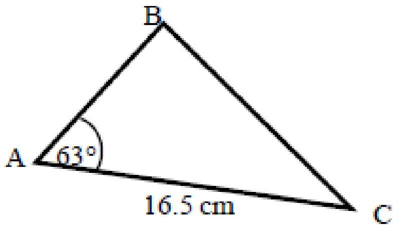
c



d



15 The area of triangle ABC is 86.7 cm^2.
Work out the length of BC.
Give your answer correct to 3 significant figures.



Answers

1	**a**	6.49 cm	**b**	6.93 cm	**c**	2.80 cm		
	d	74.3 mm	**e**	7.39 cm	**f**	6.07 cm		
2	**a**	36.9°	**b**	57.1°	**c**	47.0°	**d**	38.7°
3	5.71 cm							
4	20.4°							
5	**a**	45°	**b**	1 cm	**c**	30°	**d**	$\sqrt{3}$ cm
6	**a**	6.46 cm	**b**	9.26 cm	**c**	70.8 mm	**d**	9.70 cm
7	**a**	22.2°	**b**	52.9°	**c**	122.9°	**d**	93.6°
8	**a**	13.7 cm	**b**	76.0°				
9	**a**	4.33 cm	**b**	15.0 cm	**c**	45.2 mm	**d**	6.39 cm
10	**a**	42.8°	**b**	52.8°	**c**	53.6°	**d**	28.2°
11	**a**	8.13 cm	**b**	32.3°				
12	**a**	18.1 cm^2	**b**	18.7 cm^2	**c**	693 mm^2		
13	5.10 cm							
14	**a**	6.29 cm	**b**	84.3°	**c**	5.73 cm	**d**	58.8°
15	15.3 cm							