DIXONS SIXTH FORM ACADEMY

SUMMER WORK

A LEVEL FURTHER MATHS

2025

STUDENT NAME:

Contents

About the Summer Work

A Level Further Mathematics takes as a base your knowledge of topics from GCSE and extends these into new areas of mathematics such as complex numbers.

You need to ensure that when you start in September that you are confident on all the grade 8/9 content covered at GCSE. You will have an initial assessment in your first week covering some of the grade 8/9 content covered at GCSE.

Guidance

- Complete these tasks on A4 paper and bring them with you to your first further maths lesson.
- Each page should be labelled with the title of the task and question numbers included.
- Work should be self-marked using green pen and corrected for errors.
- Attempt every question and always show your working.
- Spend additional time on tasks you struggle with, using corbettmaths videos to help you where necessary.
- This booklet also contains significant additional information. We would encourage you to complete all the tasks including the optional ones to fully prepare for Sixth Form study.
- Use the week-by-week schedule as a guide to how much you should be aiming to complete each week.

Welcome to Further Mathematics

Subject outline

Further Mathematics has seen the largest % increase in the number of students being entered for both the AS and A-Level exams, an increase of 9.6% for A-Level Further Mathematics. The subject sharpens many key skills, including the ability to get to grips with problems, something that lies at the centre of many fields. Students who study Further Mathematics at A-Level relish a challenge and enjoy investigating different processes. Further Mathematics students will have the ability to create an interpret mathematical models of real-world phenomena, whilst also having a mastery of a wide range of mathematical techniques and methods. Outside of lesson time, students need to use their independent study time to practice maths and check through solutions from a range of resources including online retrieval practice exercises, textbook procedural practice, and past exam papers.

Further Mathematics has always been a highly valued A-Level by Universities and employers due to its complex content and the demands of the course. Further Mathematics opens pathways for students to a wide range of courses that require students to be highly numerate and strong logical thinkers. In our technology focused society, further mathematics students can often show innovation and creativity in approaching a challenge and working to find a solution, traits which are essential in the modern-day work force. This has motivated our decision to focus our optional modules on Decision Mathematics which lends itself to a more technological society.

Students will study the Edexcel Specification for A Level Further Mathematics, with students sitting the AS Level at the end of Year 1 and the full A-Level at the end of Year 2.

AS- Level:

Paper 1 (50% of the AS Level) – Core Pure Mathematics (concepts such as complex numbers, vectors, matrices, proof and calculus)

Paper 2 (50% of the AS Level) – Decision Mathematics 1 and Decision Mathematics 2 (concepts such as algorithms, Route Inspection, Game Theory, Linear Programming and Critical Path Analysis)

A Level:

Paper 1 and 2 (50% of the A Level) – Core Pure Mathematics (concepts such as complex numbers, vectors, matrices, proof, calculus, polar coordinates, hyperbolic functions)

Paper 3 (25% of the A Level) – Decision Mathematics 1 (concepts such as algorithms, Route Inspection, Linear Programming and Critical Path Analysis)

Paper 4 (25% of the A Level) – Decision Mathematics 2 (concepts such as Game Theory, Dynamic Programming, Decision Trees, Recurrence Relations and Network Flows)

Careers & Higher Education

Studying A Level Further Mathematics opens a wide array of career and higher education opportunities due to the advanced analytical and problem-solving skills acquired.

A Level Further Mathematics is a facilitating subject, meaning that is a highly respected A Level qualification. If you are interested in studying engineering, economics, mathematics, physics, statistics, actuarial science or computer science A Level Further Mathematics is recommended. If you are interested in other routes such as biochemistry, dentistry, business studies, geography or accounting some universities may list mathematics as a useful subject but not essential.

Maths careers are some of the most highly paid careers available. Research shows that on average A Level maths students earn 11% more than other students during their lifetime. Many believe that taking maths at university has limited fields since it doesn't move straight into a vocation. However, this is certainly not the case. Students who continue maths at university can move into various careers, from graduate roles within the finance industry to working in a graduate role within the civil service. Engineering has many different degree routes and courses and is one of the most popular areas that students choose to work in after university.

An example of a highly mathematical career is a data scientist. A data scientist uses statistical and computational methods to analyse large datasets, extract insights, and inform decision-making in various industries, including technology, finance, healthcare, and marketing. This role involves working with complex data, building predictive models, and communicating findings to stakeholders. A significant part of a data scientist's job is to communicate complex technical finding to non-technical stakeholders. The ability to explain mathematical concepts clearly and concisely, homed in Further Mathematics, is crucial. In addition, the role can often involve using programming languages and software tools to handle and analyse data.

Links to key information:

dixons6a.com/uploads/files/Maths.pdf

https://qualifications.pearson.com/content/dam/pdf/A%20Level/Mathematics/2017/specification-and-sample-assesment/a-level-l3-further-mathematics-specification.pdf

https://amsp.org.uk/teachers/11-16-maths/transition-to-level-3-maths/where-maths-meets-the-world-of-work/

Summer work tasks

Task 1

There are 3 practice papers to complete, with answers, that are all revision of key grade 8/9 topics from GCSE. These topics are essential to the study of A Level Further Mathematics and students need to ensure they fully understand each concept. **Students will be assessed in the first lesson on these topics** to ensure they are starting the course with a grounded understanding of algebra from GCSE. Please see the guidance on the previous page on how to set out your work. Below is a suggested week-by-week schedule to help you organize your time.

Task 2

There are several problems for you to attempt to prepare you for the problem-solving skills that will be necessary when studying A-Level Further Mathematics. Attempt all the questions to improve your problem-solving skills.

Week	Exercise
1	Paper 1 Grade 9 Questions
2	Paper 2 Grade 9 Questions
3	Paper 3 Grade 9 Questions
4	Problems 1, 2 and 3
5	Problems 4, 5 and 6
6	Problems 7 and 8

Reading list

Suggested reading:

The Codebook by *Simon Singh*

The Simpsons and Their Mathematical Secrets by *Simon Singh*

Infinity: The Quest to Think the Unthinkable by *Brian Clegg*

The Man who knew Infinity by *Robert Kanigel*

Humble Pi: A Comedy of Maths Errors by *Matt Parker*

Suggested viewing:

bbc.co.uk/iplayer/episode/b0074rxx/horizon-19951996-fermats-last-theorem

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 1/Level 2 GCSE (9-1)

Aiming for Grade 9

Paper reference **1MA1/1H**

Mathematics

PAPER 1 (Non-Calculator)
Higher Tier

42 marks 40 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, Formulae Sheet (enclosed). Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **show all your working.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.
- **Calculators may not be used.**

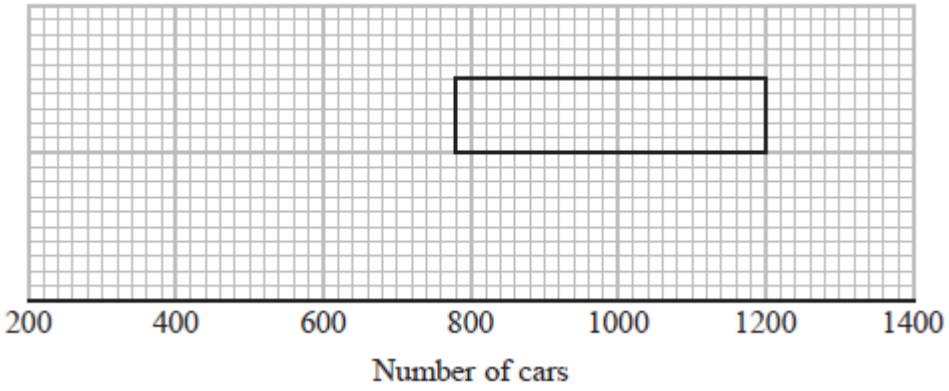
Information

- The total mark for this paper is 42. There are 10 questions.
- Questions have been broadly arranged in an ascending order of mean difficulty, as found by students achieving Grade 9 in the Summer and November 2023 examinations.
- Questions marked with an asterisk (*) also appear on the Foundation Tier paper.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Answer all questions.
Write your answers in the spaces provided.
You must write down all the stages in your working.

1 Alice recorded the number of cars going into a village on each of 80 days.
The incomplete table and the incomplete box plot give information about her results.

	Number of cars
Least number	300
Lower quartile	
Median	900
Upper quartile	
Range	1000



(*a*) (i) Use the information in the table to complete the box plot.
(ii) Use the information in the box plot to complete the table.

(3)

On some of these 80 days Alice saw fewer than 1200 cars going into the village.

(*b*) Work out an estimate for the number of days Alice saw fewer than 1200 cars going into the village.

.....

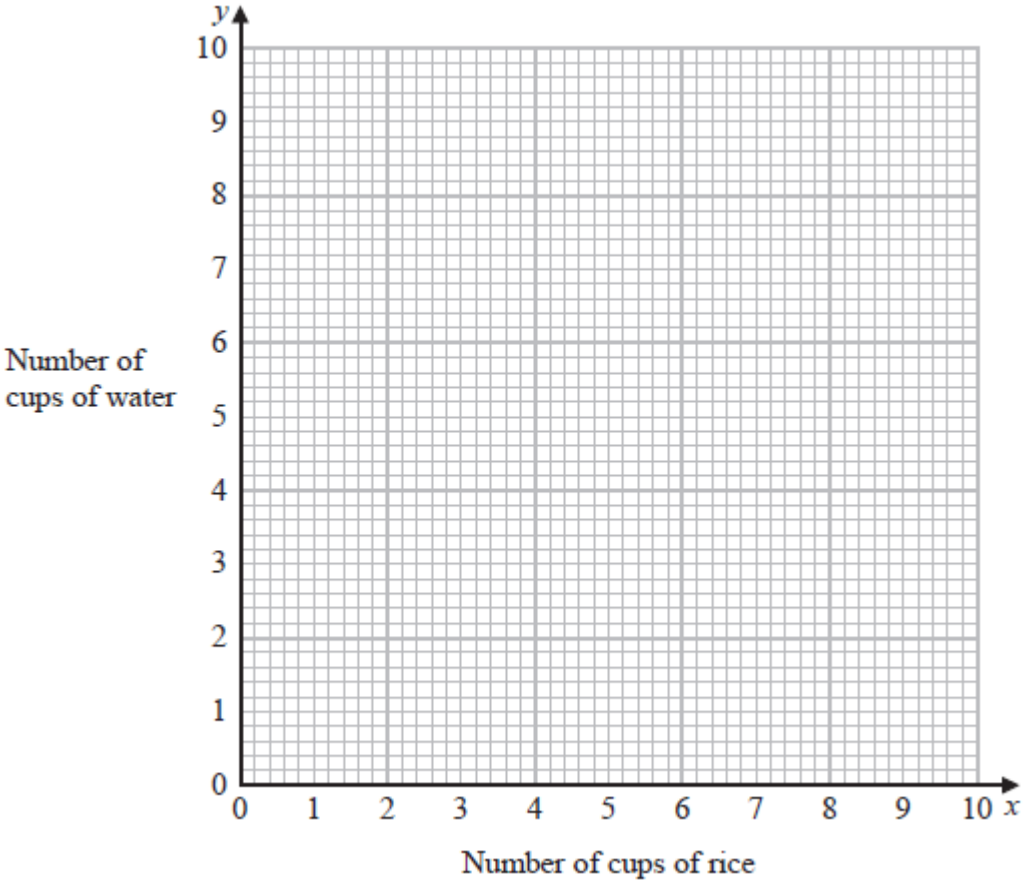
(2)

(Total for Question 1 is 5 marks)

2 To cook rice

the number of cups of rice (x) : the number of cups of water (y) = 4 : 5

(a) Use this information to draw a graph to show the relationship between the number of cups of rice and the number of cups of water needed to cook rice.



(2)

(b) (i) Find the gradient of the line drawn in part (a).

.....

(1)

(ii) Explain what this gradient represents.

.....

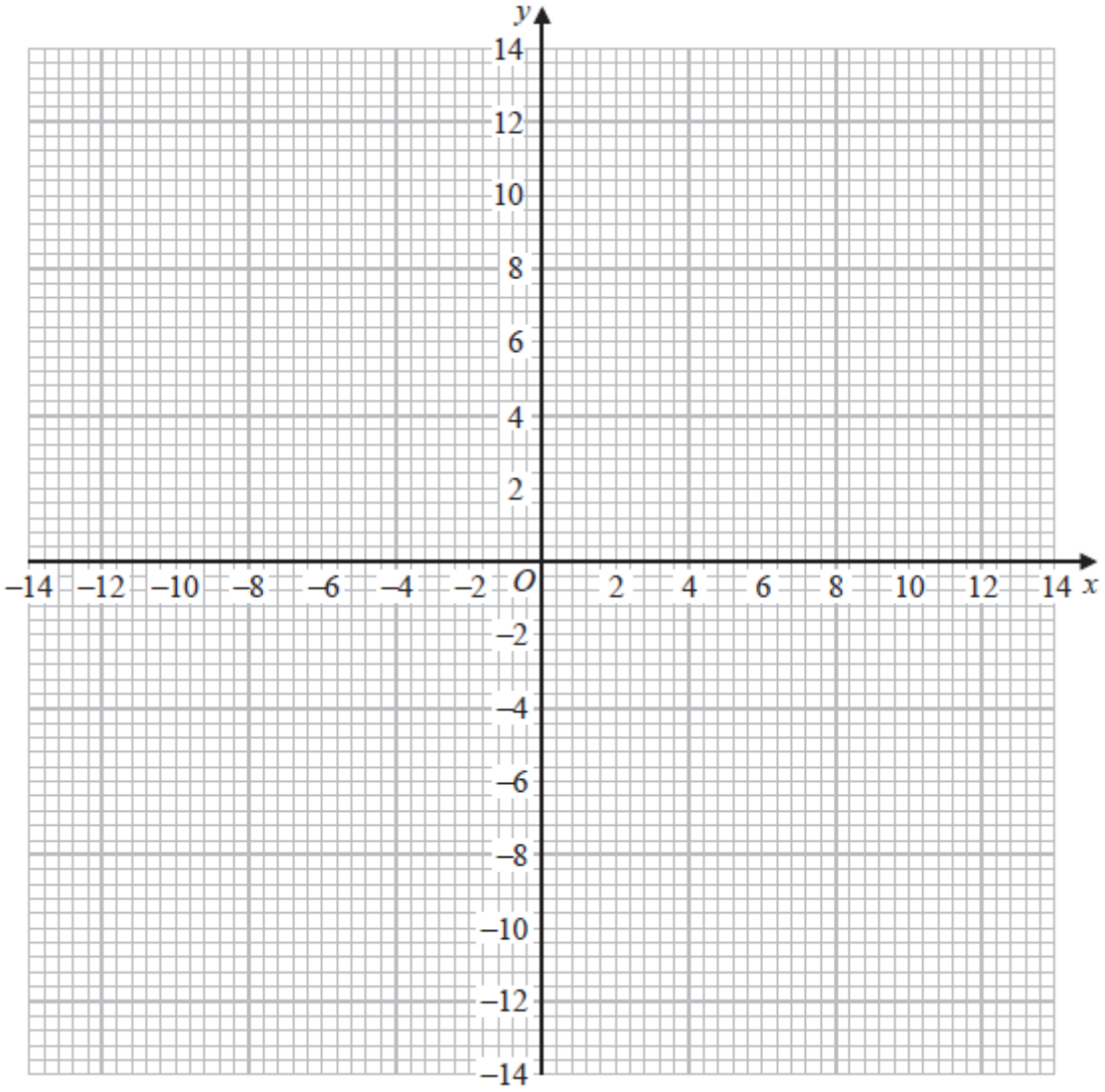
.....

.....

(1)

(Total for Question 2 is 4 marks)

3 (*a*) On the grid, draw the graph of $x^2 + y^2 = 169$



(2)

(*b*) Use your graph to find estimates for the solutions of the simultaneous equations

$$x^2 + y^2 = 169$$
$$2y = 3x$$

.....

(3)

(Total for Question 3 is 5 marks)

4 7 kg of carrots and 5 kg of tomatoes cost a total of 480p

cost of 1 kg of carrots : cost of 1 kg of tomatoes = 5 : 9

Work out the cost of 1 kg of carrots and the cost of 1 kg of tomatoes.

carrots .....p

tomatoes .....p

(Total for Question 4 is 4 marks)

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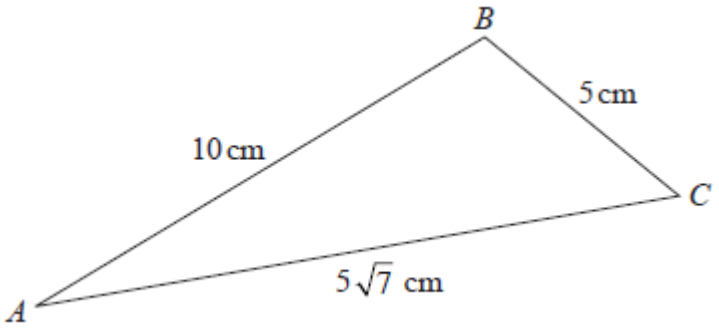
5 Write $\frac{3\sqrt{3}}{4-\sqrt{3}} - \frac{2}{\sqrt{3}}$ in the form $\frac{a\sqrt{3}+b}{c}$ where a, b and c are integers.

.....

(Total for Question 5 is 4 marks)

6 Here is triangle ABC.



Find the size of angle ABC.
You must show all your working.

.....°

(Total for Question 6 is 4 marks)

7 Solid **A** and solid **B** are similar.
The ratio of the height of solid **A** to the height of solid **B** is 2 : 5
The volume of solid **A** is 12 cm^3
Work out the volume of solid **B**.

..... cm^3

(Total for Question 7 is 3 marks)

---

8 The 2nd term of a geometric sequence is $3 + 2\sqrt{2}$
The 3rd term of the sequence is $13 + 9\sqrt{2}$
Find the value of the common ratio of the sequence.
Give your answer in the form $a + \sqrt{b}$ where a and b are integers.
You must show all your working.

.....

(Total for Question 8 is 4 marks)

9 Find the set of possible values of x for which

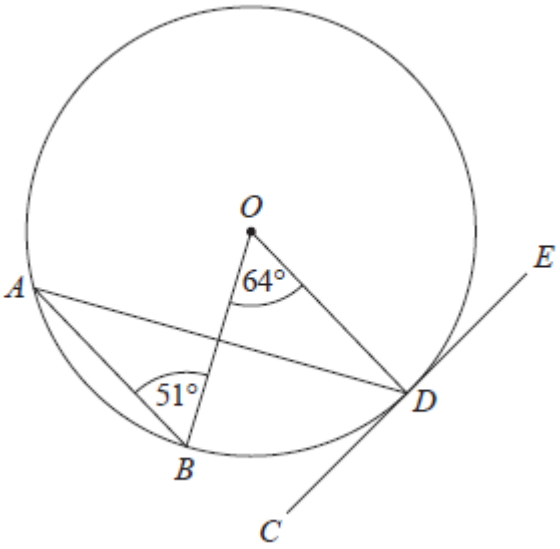
$$4x^2 - 25 < 0 \quad \textbf{and} \quad 12 - 5x - 3x^2 > 0$$

You must show all your working.

.....

(Total for Question 9 is 5 marks)

10 *A*, *B* and *D* are points on a circle with centre *O*.
CDE is the tangent to the circle at *D*.



Work out the size of angle *ADC*.
Write down any circle theorems you use.

.....°

(Total for Question 10 is 4 marks)

TOTAL FOR PAPER IS 42 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 1/Level 2 GCSE (9–1)

Aiming for Grade 9

Paper reference 1MA1/2H

Mathematics

PAPER 2 (Calculator)
Higher Tier

40 marks 40 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator, Formulae Sheet (enclosed). Tracing paper may be used.

Total Marks

Instructions

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- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **show all your working.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.

Information

- The total mark for this paper is 40. There are 12 questions.
- Questions have been broadly arranged in an ascending order of mean difficulty, as found by students achieving Grade 9 in the Summer and November 2023 examinations.
- Questions marked with an asterisk (*) also appear on the Higher Tier paper.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

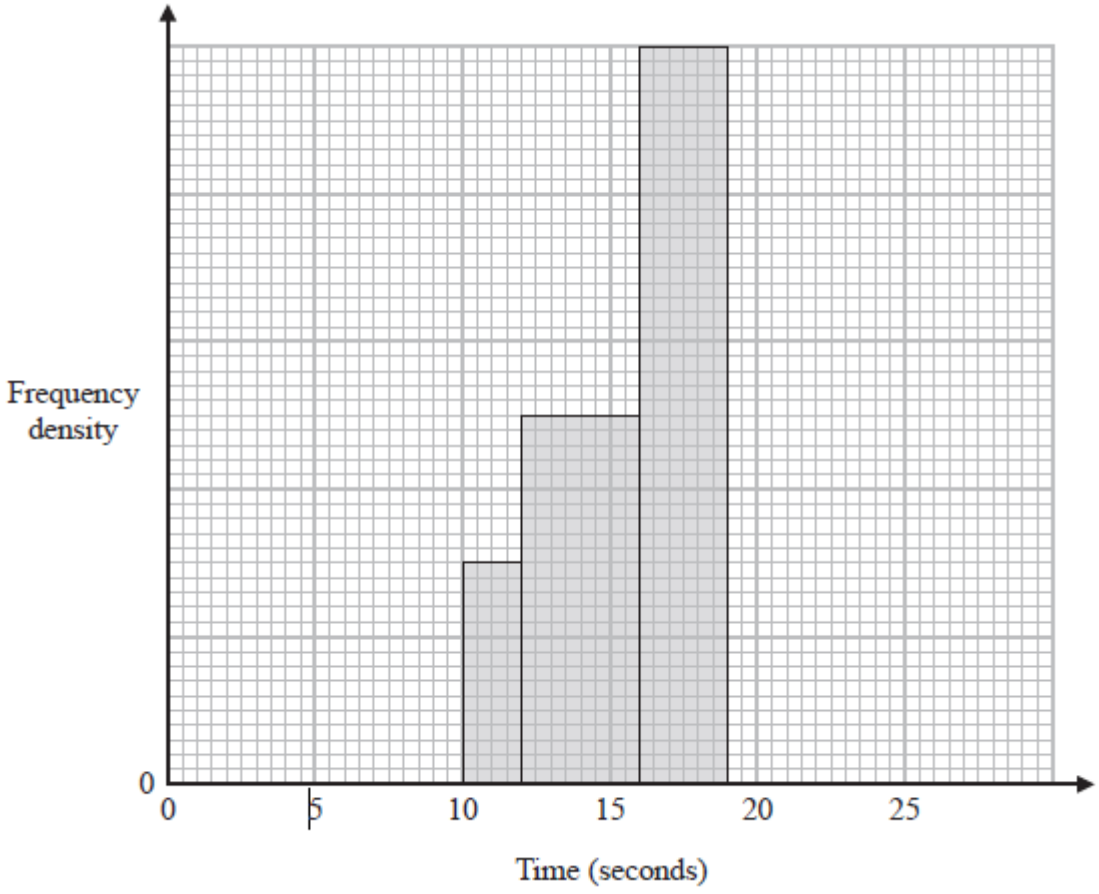
Answer all questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The incomplete table and the incomplete histogram give information about the times taken by some students to run a race.

Time (t seconds)	**Frequency**
$10 < t \leq 12$	
$12 < t \leq 16$	10
$16 < t \leq 19$	15
$19 < t \leq 21$	9
$21 < t \leq 26$	7



None of these students had a time for the race such that $t < 10$ or $t > 26$

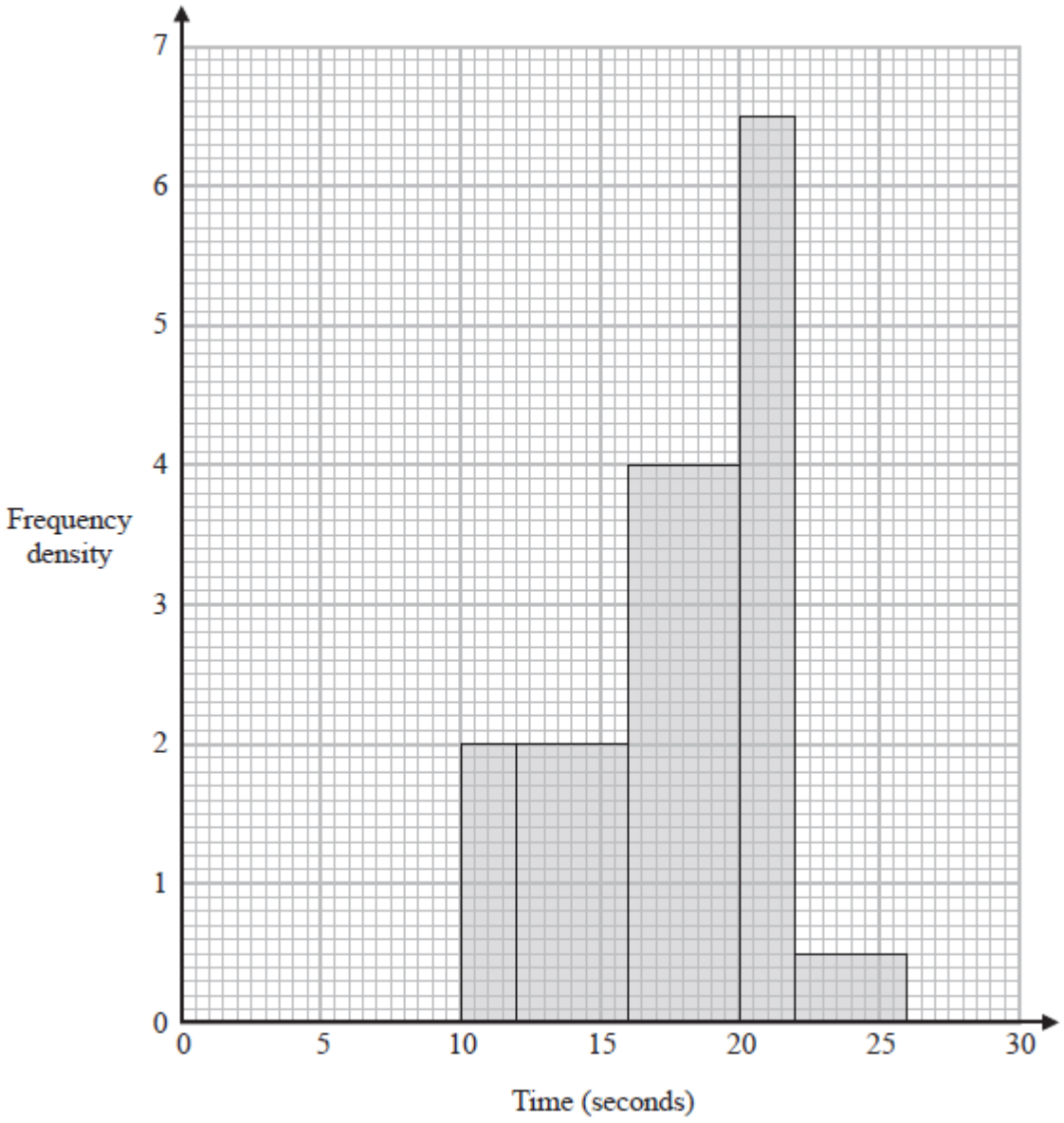
(a) Use the histogram to complete the table.

(1)

(*b*) Use the table to complete the histogram.

(2)

The histogram below gives information about the times taken by 43 students to run a different race.



(*c*) Work out an estimate for the median of the times taken by these 43 students to run the race.

..... seconds

(3)

(Total for Question 1 is 6 marks)

2 A biased dice is thrown 60 times.
The table shows information about the number that the dice lands on each time.

Number on dice	1	2	3	4	5	6
Frequency	12	7	8	9	9	15

Gethin throws the dice twice.

(*a*) Work out an estimate for the probability that the dice will land on 6 both times.

.....

(3)

Sally is going to throw the same dice *n* times and record the number it lands on each time.
She will use her results to work out a more reliable estimate for the probability in part (*a*).
(*b*) What can you say about the value of *n*?

.....

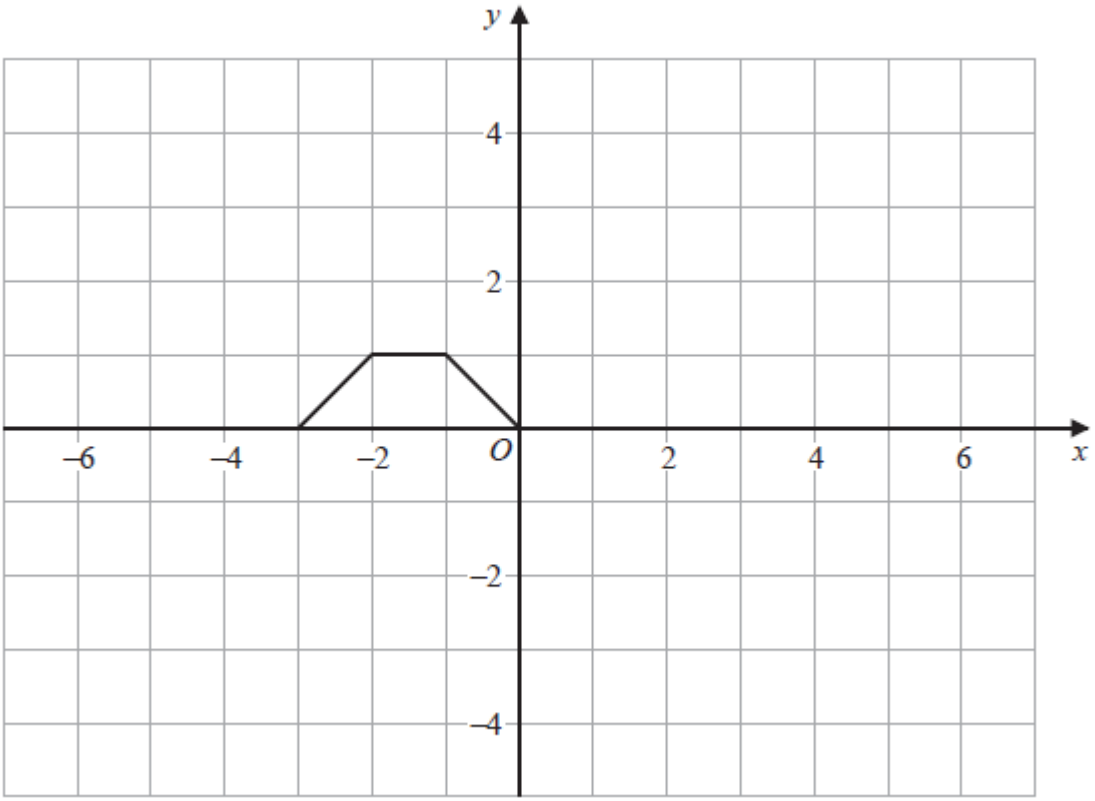
.....

.....

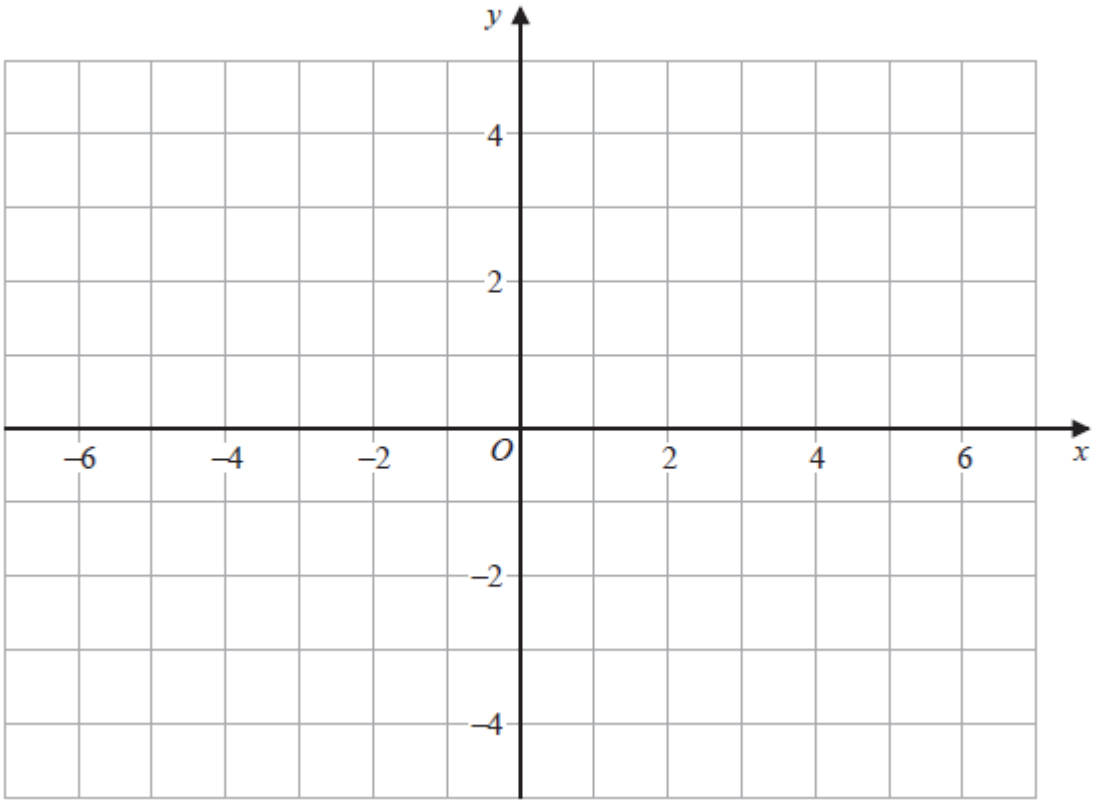
(1)

(Total for Question 2 is 4 marks)

3 Here is the graph of $y = f(x)$



On the grid below, draw the graph of $y = f(-x)$



(Total for Question 3 is 1 mark)

***4** The bearing of port B from port A is 147°
Work out the bearing of port A from port B.

.....°

(Total for Question 4 is 2 marks)

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5 $2a : 5c = 6 : 25$
$4b : 7c = 20 : 21$

Show that $a + b : b + c = 17 : 20$

(Total for Question 5 is 3 marks)

6 Write $\frac{14}{3x-21}+\left[(x+4)\div\frac{2x^2-6x-56}{2x+3}\right]$ in the form $\frac{ax+b}{cx+d}$ where a, b, c and d are integers.

.....

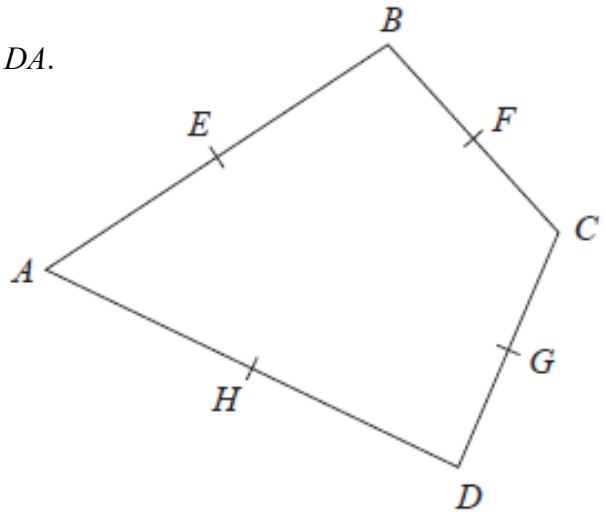
(Total for Question 6 is 4 marks)

7 $ABCD$ is a quadrilateral.

E, F, G and H are the midpoints of AB, BC, CD and DA.

$\overrightarrow{AH} = \mathbf{a}$ $\overrightarrow{AE} = \mathbf{b}$ $\overrightarrow{DG} = \mathbf{c}$

Prove, using vectors, that $EFGH$ is a parallelogram.



(Total for Question 7 is 4 marks)

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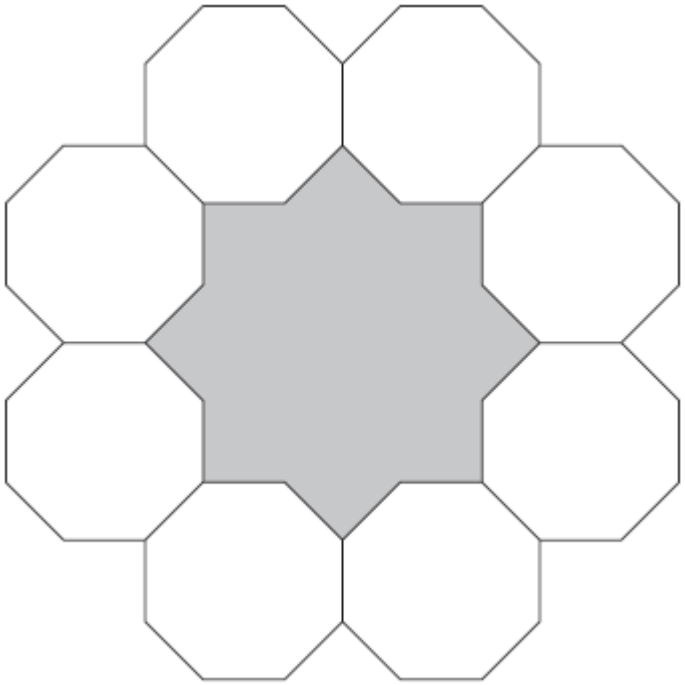
8 Show that the equation $x^3 + 2x - 6 = 0$ has a solution between $x = 1$ and $x = 2$

(Total for Question 8 is 2 marks)

9 The diagram shows 8 identical regular octagons joined to enclose a shaded shape.



Each octagon has sides of length a.
Find, in terms of a, an expression for the area of the shaded shape.
Give your answer in the form $p\left(2+\sqrt{2}\right)a^2$ where p is an integer.

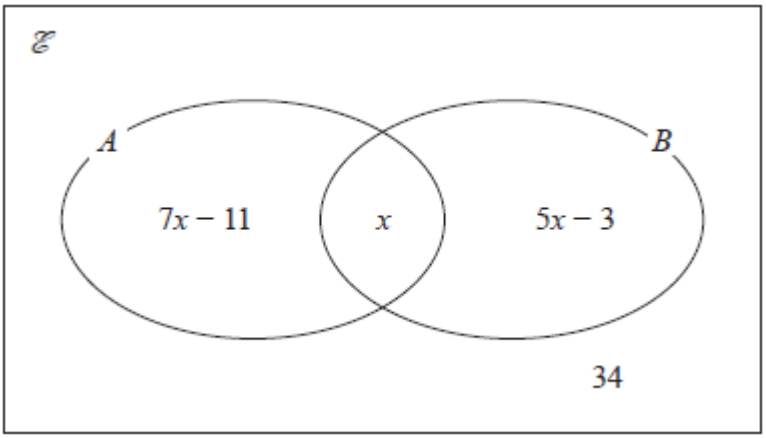
You must show all your working.

.....

(Total for Question 9 is 5 marks)

10 Vicky has a collection of medals.
The Venn diagram gives information about the number of medals in her collection where

$\mathscr{E}$ = {all medals}
A = {English medals}
B = {gold medals}



Vicky is going to take at random a medal from her collection.

Given that the medal is gold, the probability that the medal is English is $\frac{2}{11}$

Work out the number of medals in Vicky's collection.

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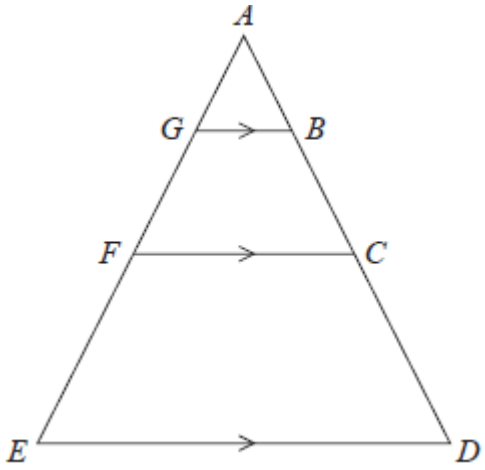
(Total for Question 10 is 4 marks)

11 Here are three similar triangles, *ABG*, *ACF* and *ADE*.



ABCD and *AGFE* are straight lines.
AB : *BC* : *CD* = 1 : 2 : 3
Show that

area of *ABG* : area of *BCFG* : area of *CDEF* = 1 : 8 : 27

(Total for Question 11 is 3 marks)

12 There are only blue pens and red pens in a box.
The number of blue pens is four times the number of red pens.
Rita takes at random one pen from the box.
She records the colour of the pen and then replaces it in the box.
Rita does this n times, where $n \geq 2$
Write down an expression, in terms of n, for the probability that Rita gets a blue pen at least once and a red pen at least once.

.....

(Total for Question 12 is 2 marks)

TOTAL FOR PAPER IS 36 MARKS

Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | Candidate Number

Pearson Edexcel Level 1/Level 2 GCSE (9–1)

Aiming for Grade 9

Paper reference **1MA1/3H**

Mathematics

PAPER 3 (Calculator)
Higher Tier
37 marks 40 minutes

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser, calculator, Formulae Sheet (enclosed). Tracing paper may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
-
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You must **show all your working.**
- Diagrams are **NOT** accurately drawn, unless otherwise indicated.

Information

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- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Answer all questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (*a*) Factorise $a^2 - b^2$

.....

(1)

(*b*) Show that $2^{40} - 1$ is the product of two consecutive odd numbers.

(2)

(Total for Question 1 is 3 marks)

2 A circle has equation $x^2 + y^2 = 25$

The point P with coordinates $(-3, 4)$ lies on the circle.
Alex says that the tangent to the circle at P crosses the x-axis at the point $(-8, 0)$

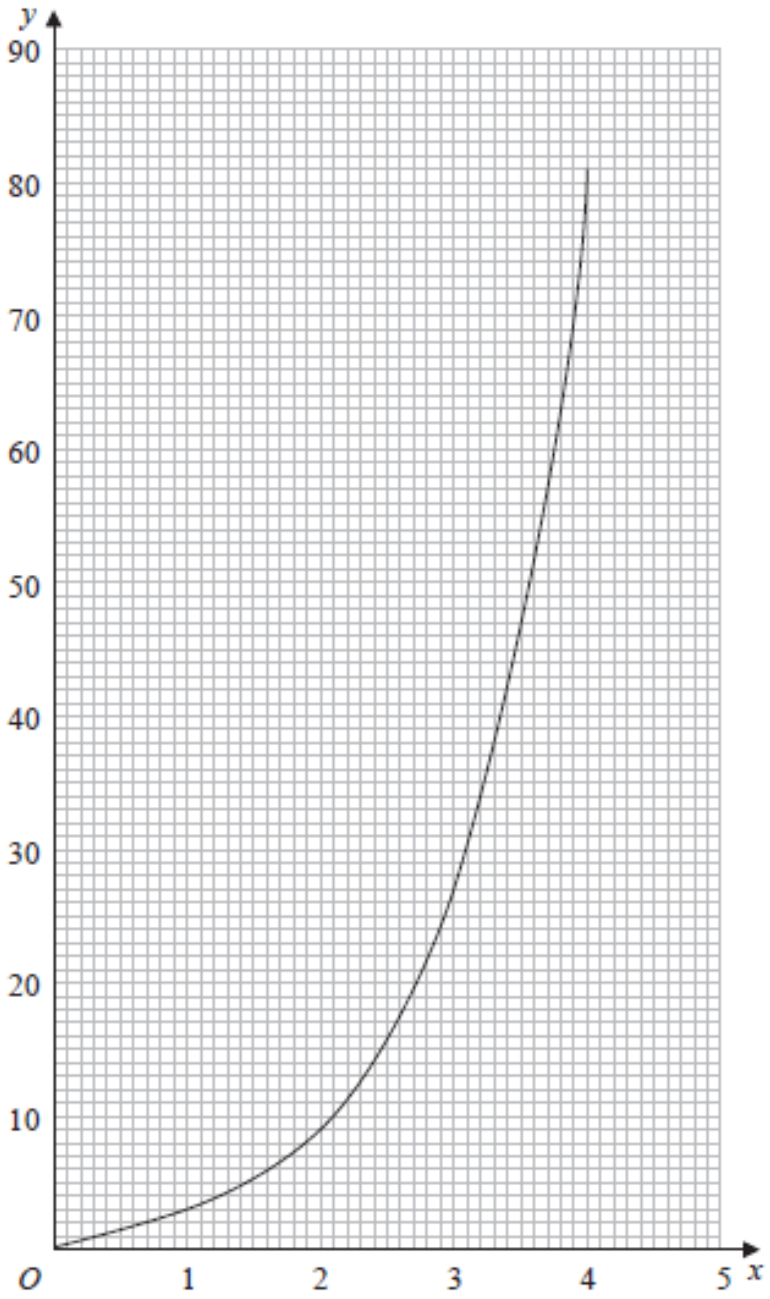
Is Alex correct?

You must show how you get your answer.

(Total for Question 2 is 4 marks)

3 Sana needs to draw the graph of $y = 3^x$ for $0 \leq x \leq 4$

She draws the graph shown on the grid.



Write down one thing Sana has done wrong.

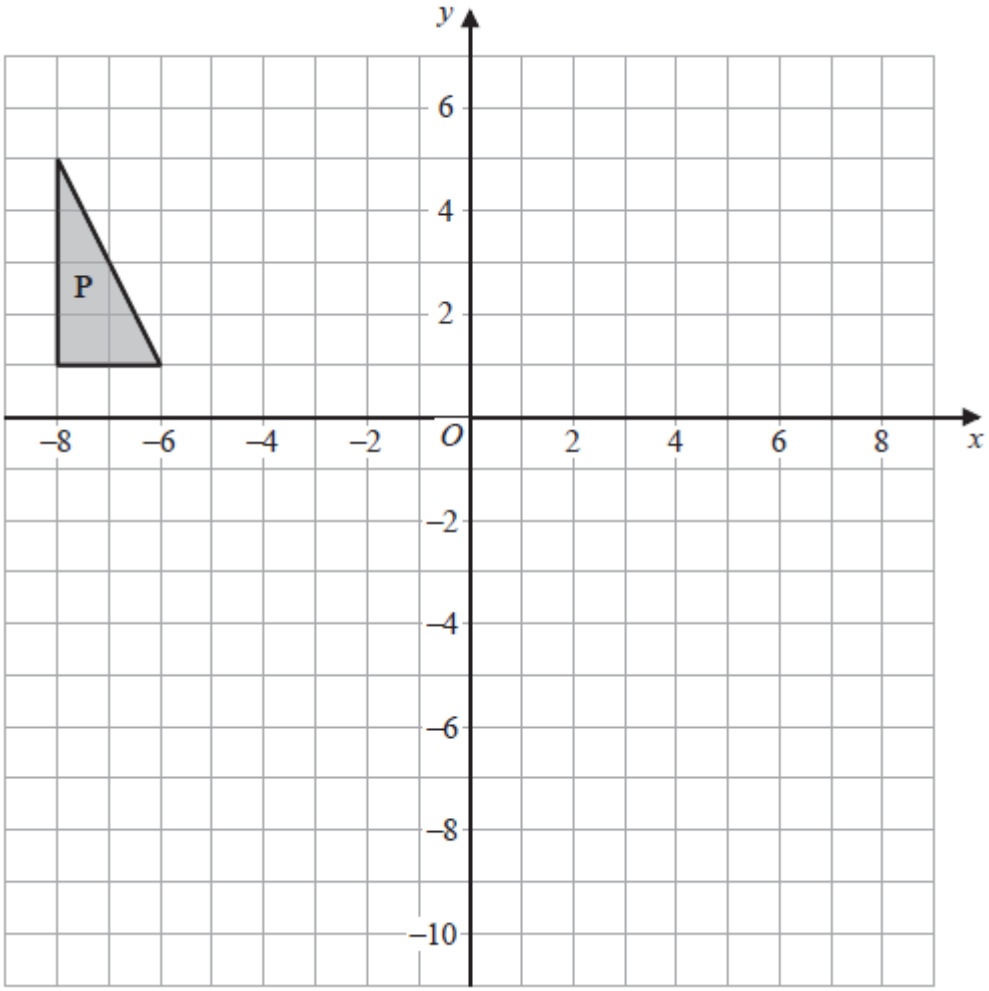
.....

.....

.....

(Total for Question 3 is 1 mark)

4



(*a*) Enlarge triangle **P** by scale factor $-1\frac{1}{2}$ with centre of enlargement (–2, –1)

Label your image **Q**.

(2)

Triangle **P** is transformed by a combined transformation of a rotation of 90° anticlockwise about the origin followed by a translation to give triangle **R**.
Exactly one vertex of triangle **P** is invariant under the combined transformation.

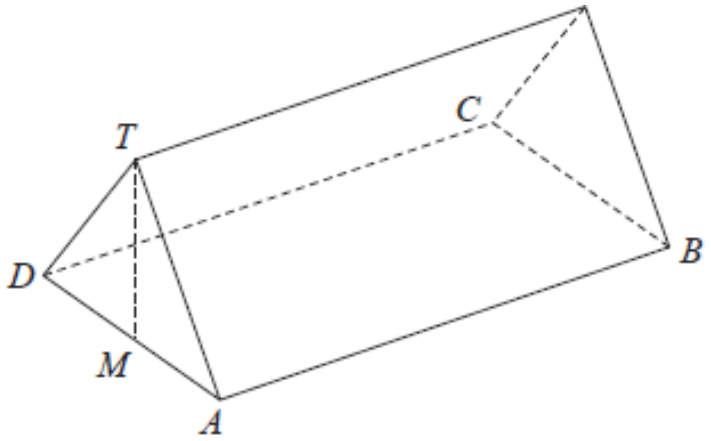
(*b*) Find one possible column vector for the translation.

$$\begin{pmatrix} \ldots\ldots \\ \ldots\ldots \end{pmatrix}$$

(1)

(Total for Question 4 is 3 marks)

5 The diagram shows a triangular prism with a horizontal rectangular base $ABCD$.



M is the midpoint of AD.

The vertex T of the prism is vertically above M.

$AB = 14.7$ cm $\quad BC = 3.8$ cm $\quad MT = 2.3$ cm

P is the point on AB such that

$$AP : PB = 5 : 2$$

Calculate the size of the angle between TP and the base $ABCD$ of the prism.
Give your answer correct to 1 decimal place.

.....°

(Total for Question 5 is 4 marks)

6 An expression for the *n*th term of the sequence of triangular numbers is $\frac{n(n+1)}{2}$

Prove that the sum of any two consecutive triangular numbers is a square number.

(Total for Question 6 is 3 marks)

---

7 The floor plan of a house is drawn using a scale of 1 : 50

On the plan, a room in the house has a floor area of 48 cm^2

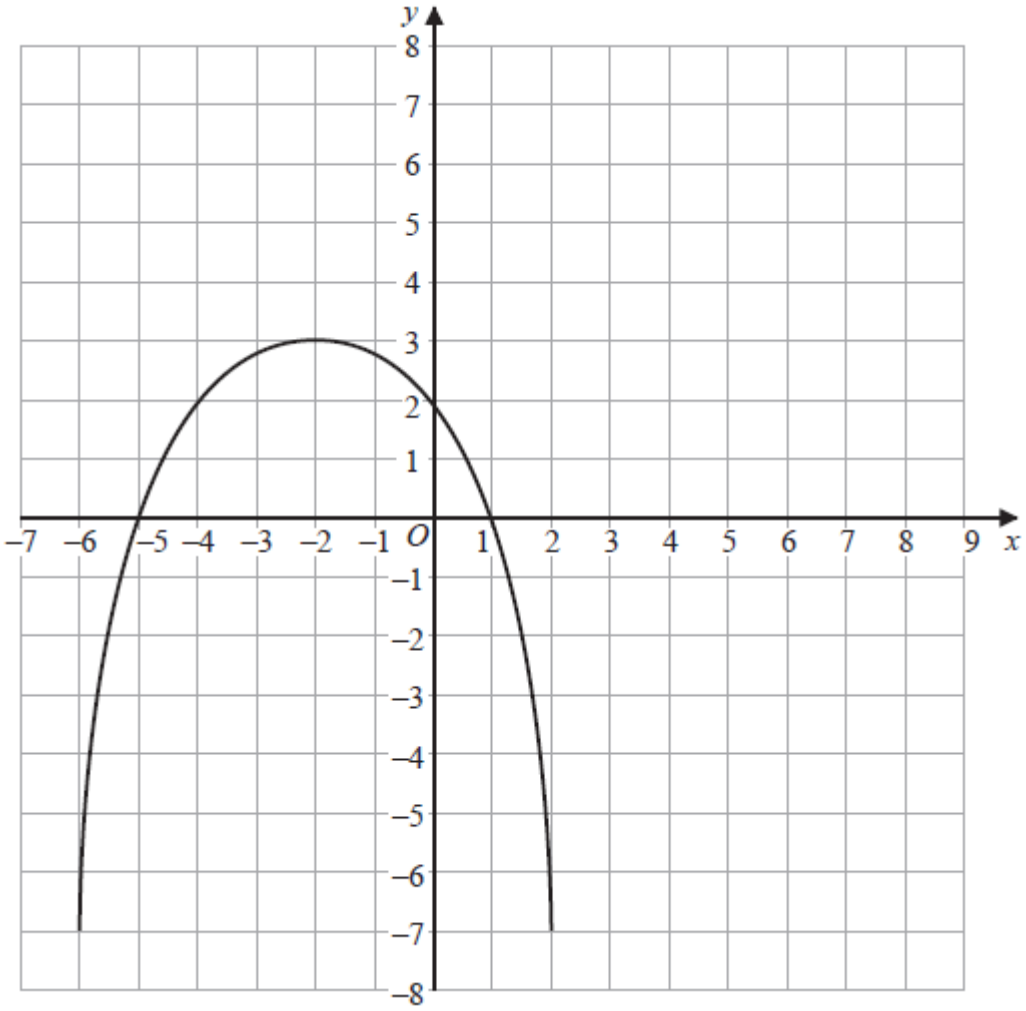
Work out the real area of the floor of this room.

Give your answer in m^2

..... m^2

(Total for Question 7 is 3 marks)

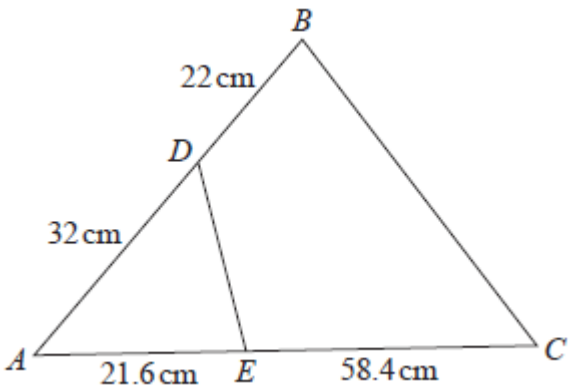
8 The graph of $y = f(x)$ is shown on the grid.



On the grid, sketch the graph of $y = f(-x) + 3$

(Total for Question 8 is 2 marks)

9 The diagram shows triangle *ABC* and triangle *AED*.



Show that triangle *ABC* and triangle *AED* are similar.

(Total for Question 9 is 2 marks)

10 There is a total of y counters in a box.

There are x pink counters and 5 blue counters in the box.

The rest of the counters are green.

$$x : y = 1 : 3$$

Freda takes at random two counters from the box.

Find, in terms of x, an expression for the probability that Freda takes two counters of the same colour.

Give your answer as a fraction in the form $\frac{ax^2 + bx + c}{dx^2 + ex}$ where a, b, c, d and e are integers.

.....

(Total for Question 10 is 5 marks)

11 Ebony makes some bracelets to sell.
The materials to make all the bracelets cost £190, correct to the nearest £5

Ebony sells all the bracelets for a total of £875, correct to the nearest £5
The total time taken to make and sell all these bracelets was 72 hours, correct to the nearest hour.
Ebony uses this method to calculate her hourly rate of pay

$$\text{Hourly rate of pay} = \frac{\text{total selling price} - \text{total cost of materials}}{\text{total time taken}}$$

The minimum hourly rate of pay for someone of Ebony's age is £8.20

By considering bounds, determine if Ebony's hourly rate of pay was definitely more than £8.20

You must show all your working.

(Total for Question 11 is 4 marks)

12 Given that the vector $a\begin{pmatrix}2\\6\end{pmatrix}+b\begin{pmatrix}8\\2\end{pmatrix}$ is parallel to the vector $\begin{pmatrix}13\\6\end{pmatrix}$

find an expression for b in terms of a.

.....

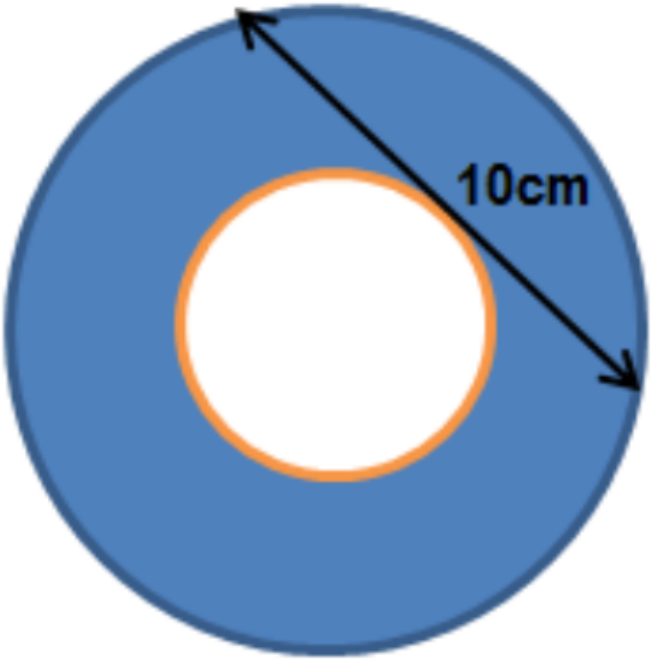
(Total for Question 12 is 3 marks)

TOTAL FOR PAPER IS 37 MARKS

Task 2

Problem 1:

Can you work out the shaded area in the diagram (the line shown just touches the smaller circle)?



Problem 2:

Find the value of

$$\frac{99}{100} \times \frac{80}{81} \times \frac{63}{64} \times \frac{48}{49} \times \frac{35}{36} \times \frac{24}{25} \times \frac{15}{16} \times \frac{8}{9} \times \frac{3}{4}.$$

Write your answer in the form $\frac{a}{b}$, where a and b are positive integers with no common factors other than 1.

Problem 3:

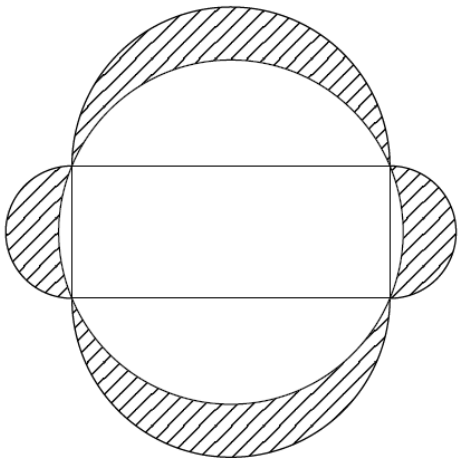
A point E lies outside the rectangle $ABCD$ such that CBE is an equilateral triangle. The area of the pentagon $ABECD$ is five times the area of the triangle CBE.

What is the ratio of the lengths $AB : AD$?

Write your answer in the form $a : 1$.

Problem 4:

Four semicircles are drawn on the sides of a rectangle with width 10 cm and length 24 cm. A circle is drawn that passes through the four vertices of the rectangle.



What is the value, in cm^2, of the shaded area?

Problem 5:

The points $A(1,2)$ and $B(-2,1)$ are two vertices of a rectangle $ABCD$. The diagonal CA produced passes through the point $(2,9)$. Calculate the coordinates of the vertices C and D.

Problem 6:

(a) Which positive integer in the range from 1 to 250 has more different prime divisors than any other integer in this range? [3 marks]

(b) When $n = 5$ the product $n(n + 1)(n + 2)$ can be written as the product of four distinct primes. Indeed, when $n = 5$

$$n(n + 1)(n + 2) = 5 \times 6 \times 7 = 2 \times 3 \times 5 \times 7.$$

What is the least positive integer n such that $n(n+1)(n+2)$ can be written as a product of *five* distinct primes? [3 marks]

Problem 7:

Powerful quadratics

Problem

(i) Find all real solutions of the equation

$$(x^2 - 7x + 11)^{(x^2-11x+30)} = 1.$$

(ii) Find all real solutions of the equation

$$(2 - x^2)^{(x^2-3\sqrt{2}x+4)} = 1.$$

Problem 8:

Scary sum

Problem

Evaluate the sum

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + \cdots + \frac{1}{\sqrt{15}+\sqrt{16}}.$$

(You might want to use a calculator to get an estimate of the answer, but in order to get the exact answer you will have to do it by hand!)

Can you find a similar sum that evaluates to 5?

Can you find a similar sum that evaluates to a number that is not an integer?

Paper 1 Solutions

Answer all questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Alice recorded the number of cars going into a village on each of 80 days.
The incomplete table and the incomplete box plot give information about her results.

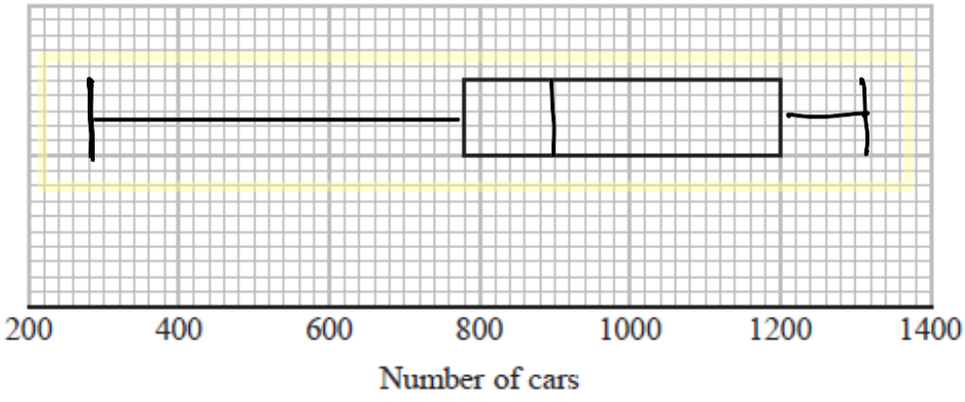
	Number of cars
Least number	300
Lower quartile	780
Median	900
Upper quartile	1200
Range	1000

Largest = 300 + 1000 = 1300

3 marks for 780, 1200, median plotted and both whiskers drawn

2 marks for two of the above

1 mark for any one of the above



(a) (i) Use the information in the table to complete the box plot.
(ii) Use the information in the box plot to complete the table.

(3)

On some of these 80 days Alice saw fewer than 1200 cars going into the village.

(b) Work out an estimate for the number of days Alice saw fewer than 1200 cars going into the village.

1200 → $\frac{3}{4}$ (75%)

$\frac{3}{4} \times 80 = \frac{80}{4} \times 3$

$= 60$

1 mark

60 Final mark

.....

(2)

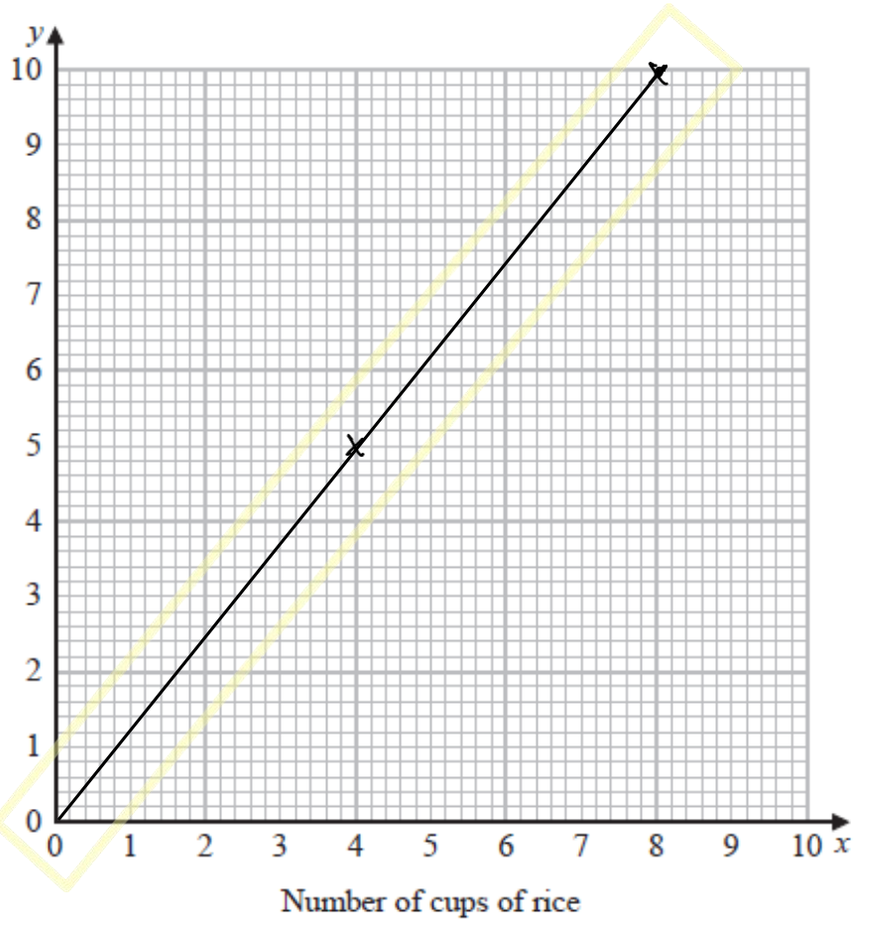
(Total for Question 1 is 5 marks)

2 To cook rice

the number of cups of rice (x) : the number of cups of water (y) = 4 : 5

(a) Use this information to draw a graph to show the relationship between the number of cups of rice and the number of cups of water needed to cook rice.

$x : y$
$4 : 5$
$= 8 : 10$



2 marks for correct line drawn

1 mark for plotting two correct points on the line

(2)

(b) (i) Find the gradient of the line drawn in part (*a*).

$\frac{\text{change in } y}{\text{change in } x} = \frac{10}{8} = \frac{5}{4} = 1.25$

1.25 1 mark

.....

(1)

(ii) Explain what this gradient represents.

The number of cups of water for each cup of rice 1 mark

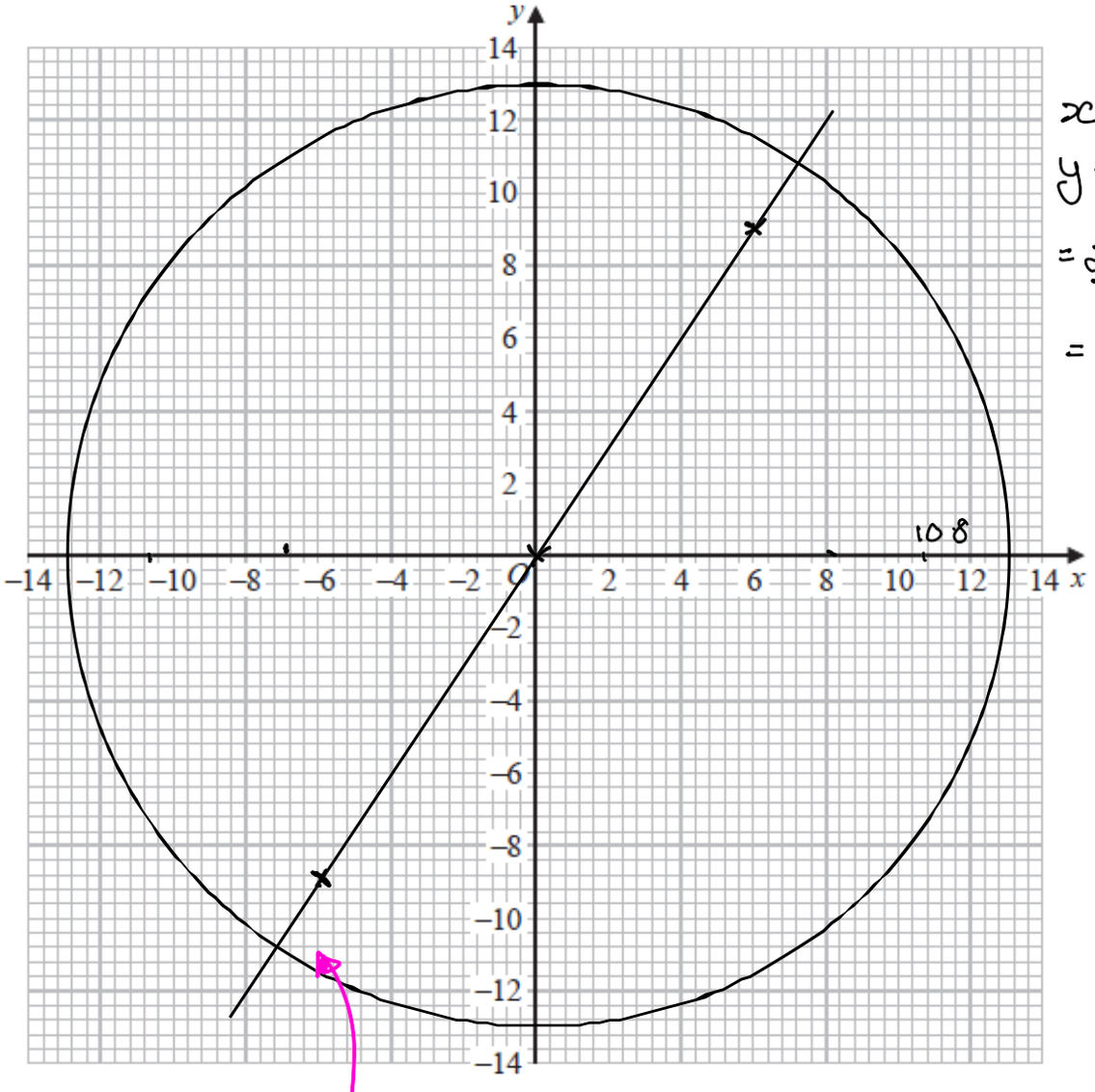
.....

.....

(1)

(Total for Question 2 is 4 marks)

3 (*a*) On the grid, draw the graph of $x^2 + y^2 = 169$



$x = 10{\cdot}8$

$y = \frac{2}{3} 10{\cdot}8$

$= \frac{21{\cdot}6}{3}$

$= 7{\cdot}2$

2 marks for correct circle drawn **(2)**

(*b*) Use your graph to find estimates for the solutions of the simultaneous equations

$x^2 + y^2 = 169$

$2y = 3x$

$2y = 3x$

$y = \frac{3}{2} x$

1 mark for correct line drawn

$x = 0 \quad y = 0$

$x = -6 \quad y = -9$

$x = 6 \quad y = 9$

Final 2 marks for correct values given as pairs

$x = 7{\cdot}2 \quad y = 10{\cdot}8$

$x = -7{\cdot}2 \quad y = -10{\cdot}8$

.....

(3)

(Total for Question 3 is 5 marks)

1 mark may be awarded for both correct x values, both correct y values or one correct pair if two marks is not awarded

4 7 kg of carrots and 5 kg of tomatoes cost a total of 480p

cost of 1 kg of carrots : cost of 1 kg of tomatoes = 5 : 9

Work out the cost of 1 kg of carrots and the cost of 1 kg of tomatoes.

check.
$7\times30 + 5\times54 = 210+270 = 480$ ✓

carrots : toms

5 : 9

$\frac{t}{c} = \frac{9}{5}$

$t = \frac{9}{5}c$ 1 mark

1 mark

$7c + 5t = 480$

$7c + 5(\frac{9}{5}c) = 480$

$16c = 480$ 1 mark

$c = \frac{480}{16} = 30$

$t = \frac{9}{5}\times30 = \frac{270}{5}$

$= 54$

Final mark

carrots ..... 30 ..... p

tomatoes ..... 54 ..... p

(Total for Question 4 is 4 marks)

---

5 Write $\frac{3\sqrt{3}}{4-\sqrt{3}} - \frac{2}{\sqrt{3}}$ in the form $\frac{a\sqrt{3}+b}{c}$ where *a*, *b* and *c* are integers.

1 mark

$\frac{3\sqrt{3}}{4-\sqrt{3}} \times \frac{4+\sqrt{3}}{4+\sqrt{3}} = \frac{12\sqrt{3} + 3\sqrt{3}\sqrt{3}}{16 + 4\sqrt{3} - 4\sqrt{3} - 3} = \frac{12\sqrt{3}+9}{13}$

1 mark

$\frac{2}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$

1 mark

so: $\frac{12\sqrt{3}+9}{13} - \frac{2\sqrt{3}}{3} = \frac{3(12\sqrt{3}+9) - 13\times2\sqrt{3}}{39}$

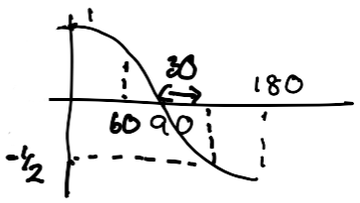
$= \frac{36\sqrt{3}+27-26\sqrt{3}}{39}$

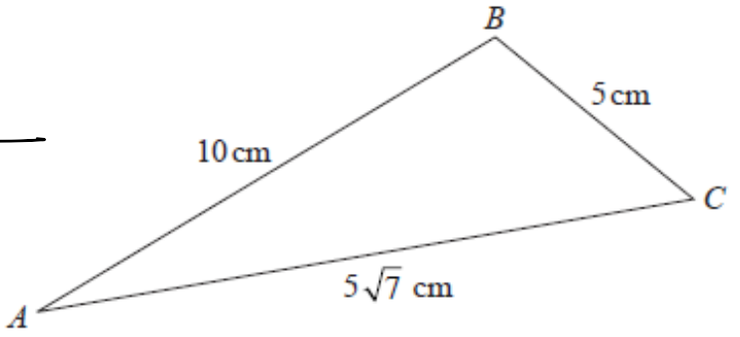
$\frac{10\sqrt{3}+27}{39}$ Final mark

.....

(Total for Question 5 is 4 marks)

6 Here is triangle *ABC*.





Find the size of angle *ABC*.

You must show all your working.

$c^2 = a^2 + b^2 - 2ab\cos C$

$(5\sqrt{7})^2 = 10^2 + 5^2 - 2 \times 10 \times 5 \times \cos B$ 1 mark

$\cos B = \dfrac{5^2 + 10^2 - (5\sqrt{7})^2}{2 \times 5 \times 10}$ 1 mark

$= \dfrac{125 - 175}{100}$

$= \dfrac{-50}{100} = -\dfrac{1}{2}$ 1 mark

So $90 + 30 = 120$

(seeing graph at the top)

120 Final mark

..... °

(Total for Question 6 is 4 marks)

7 Solid **A** and solid **B** are similar.

The ratio of the height of solid **A** to the height of solid **B** is 2 : 5

The volume of solid **A** is 12 cm^3

Work out the volume of solid **B**.

	A		B
height	2	:	5
length SF		$\times \frac{5}{2}$	
Volume	12	:	?

1 mark

$12 \times \left(\frac{5}{2}\right)^3$

1 mark

$= \frac{12 \times 125}{8_2} = \frac{375}{2}$

Final mark

187.5 ..... cm^3

(Total for Question 7 is 3 marks)

---

8 The 2nd term of a geometric sequence is $3 + 2\sqrt{2}$

The 3rd term of the sequence is $13 + 9\sqrt{2}$

Find the value of the common ratio of the sequence.

Give your answer in the form $a + \sqrt{b}$ where a and b are integers.

You must show all your working.

$\frac{13+9\sqrt{2}}{3+2\sqrt{2}}$

1 mark

$= \frac{13+9\sqrt{2}}{3+2\sqrt{2}} \times \frac{3-2\sqrt{2}}{3-2\sqrt{2}}$

1 mark

$= \frac{39 - 26\sqrt{2} + 27\sqrt{2} - 18\sqrt{2}\sqrt{2}}{9 - 6\sqrt{2} + 6\sqrt{2} - 4\sqrt{2}\sqrt{2}}$

1 mark

$= \frac{39 + \sqrt{2} - 36}{9 - 8}$

$= \frac{3 + \sqrt{2}}{1}$

$3\sqrt{2}$

Final mark

.....

(Total for Question 8 is 4 marks)

9 Find the set of possible values of x for which

$$4x^2 - 25 < 0 \quad \textbf{and} \quad 12 - 5x - 3x^2 > 0$$

You must show all your working.

$(2x - 5)(2x + 5) < 0$ 1 mark

$\frac{5}{2}$ $\frac{-5}{2}$

2.5 -2.5 1 mark

$-3x^2 - 5x + 12 > 0$ -36 $4, 9$

$-3x^2 - 9x + 4x + 12 > 0$

$-3x(x + 3) + 4(x + 3) > 0$

$(-3x + 4)(x + 3) > 0$ 1 mark

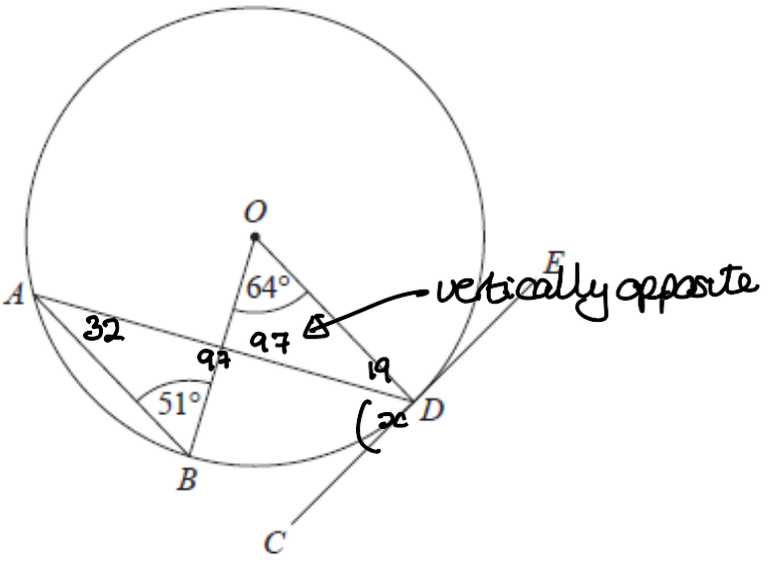
$\frac{4}{3}$ $\frac{1}{3}$ -3 1 mark

$x < \frac{4}{3}$ $x > -3$ so -2.5 OR $-\frac{5}{2}$

Final mark $-2.5 < x < \frac{4}{3}$

(Total for Question 9 is 5 marks)

10 A, B and D are points on a circle with centre O.
CDE is the tangent to the circle at D.



Work out the size of angle ADC.
Write down any circle theorems you use.

let ADC = x

1 mark BAD = 32 angle at the centre is twice the (BOD) angle at the circumference (BAD)

angle in a triangle = 180

180 − (51 + 32) = 180 − 83 = 97

1 mark

ODA = 180 − (97 + 64) = 161 = 19

x = 90 − 19 = 71 angle between tangent and radius = 90°

Final mark awarded for correct reasons

1 mark

71

Paper 2 Solutions

Answer all questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 The incomplete table and the incomplete histogram give information about the times taken by some students to run a race.

Freq. density

Time (t seconds)	Frequency
$10 < t \leq 12$	3
$12 < t \leq 16$	10
$16 < t \leq 19$	15
$19 < t \leq 21$	9
$21 < t \leq 26$	7

1 mark

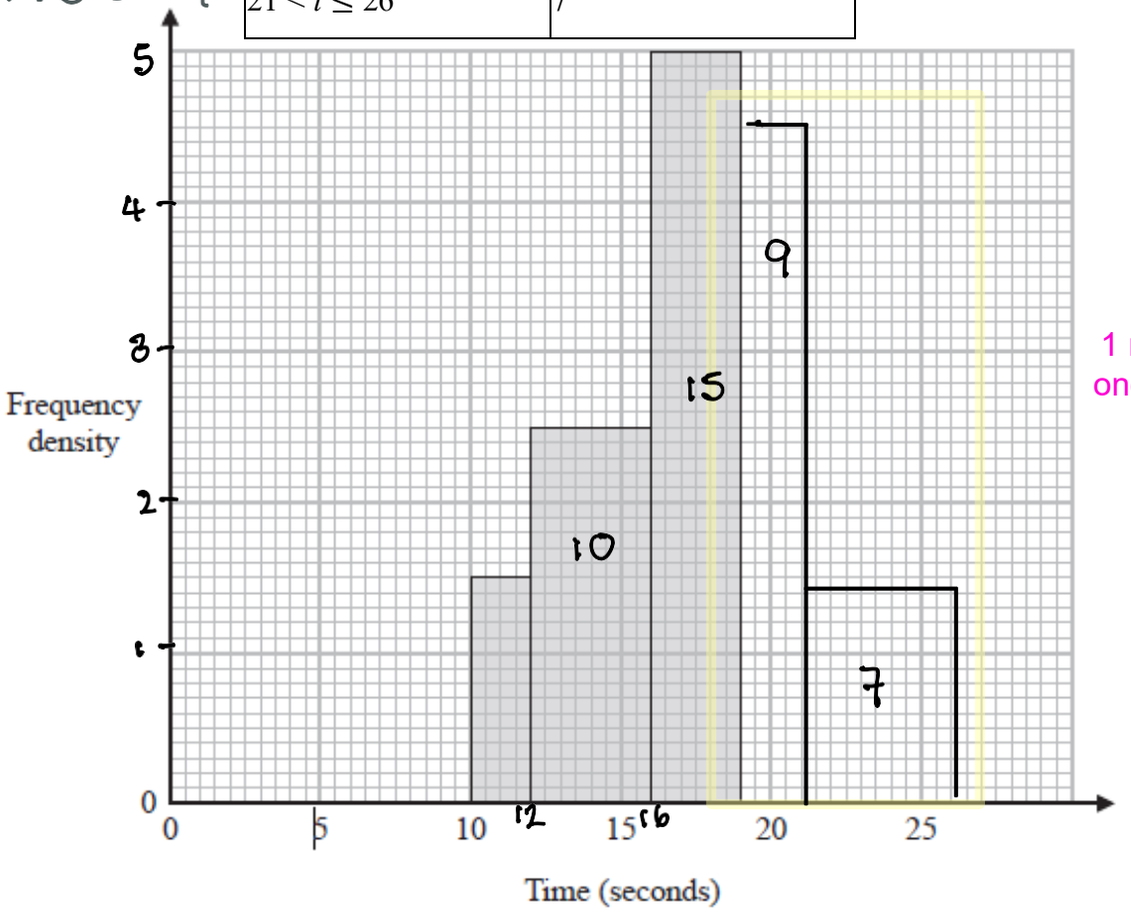
$1.5 \times 2 = 3$

$10 \div 4 = 2.5$

$15 \div 3 = 5$

$9 \div 2 = 4.5$

$7 \div 5 = 1.4$



1 mark for one correct bar

None of these students had a time for the race such that t < 10 or $t > 26$

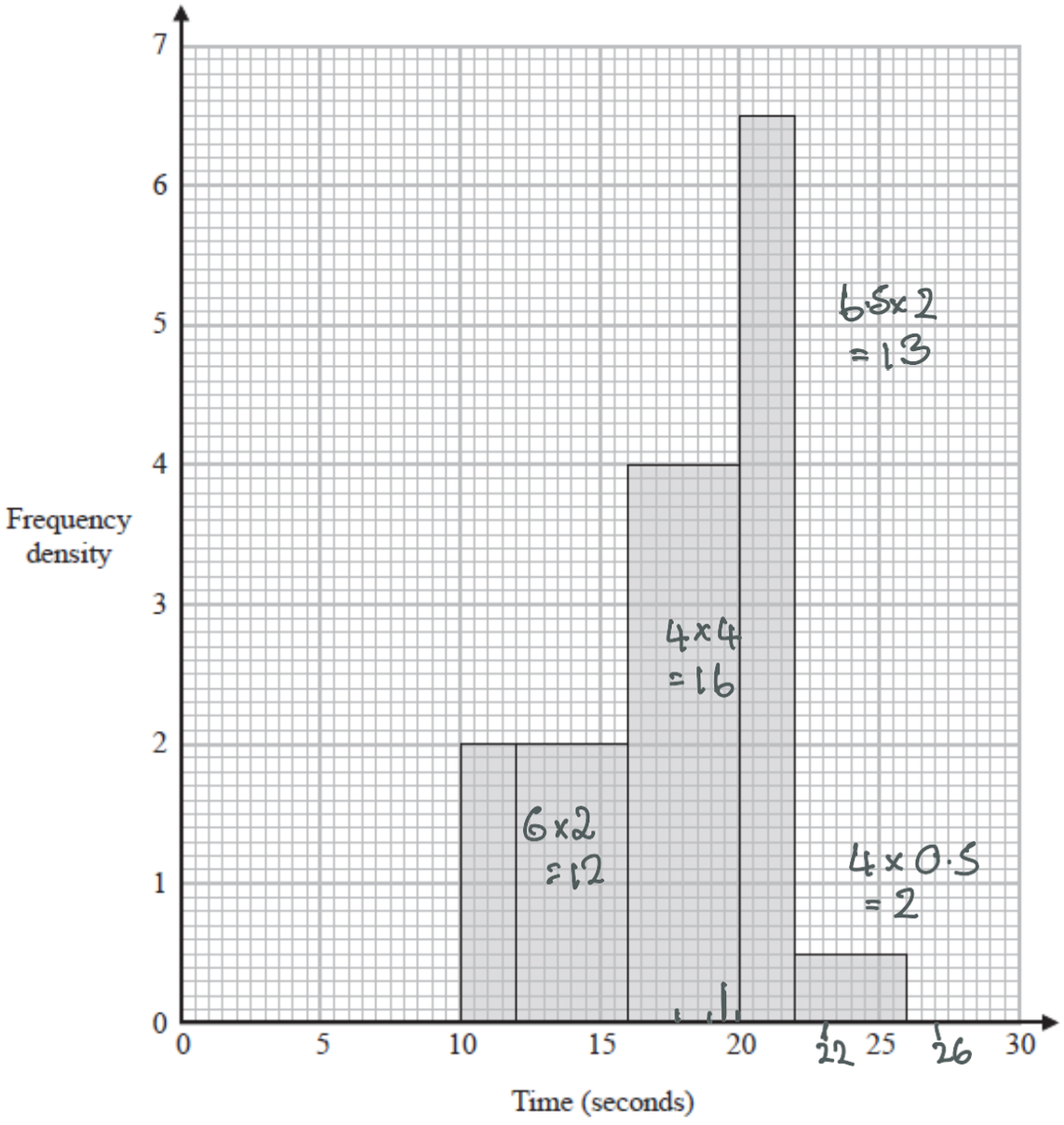
(a) Use the histogram to complete the table.

(1)

(b) Use the table to complete the histogram.

(2)

The histogram below gives information about the times taken by 43 students to run a different race.



(c) Work out an estimate for the median of the times taken by these 43 students to run the race.

1 mark

Total = 12 + 16 + 13 + 2 = 43

median = $\frac{43+1}{2}$ = 22nd

Final mark

12 = 10

18·5 .....seconds

(3)

1 mark

(Total for Question 1 is 6 marks)

2 A biased dice is thrown 60 times.
The table shows information about the number that the dice lands on each time.

Number on dice	1	2	3	4	5	6
Frequency	12	7	8	9	9	15

Gethin throws the dice twice.

(a) Work out an estimate for the probability that the dice will land on 6 both times.

1 mark $P(6) = \frac{15}{60} = \frac{1}{4}$

$P(6,6) = \frac{1}{4} \times \frac{1}{4} = \frac{1}{16}$

1 mark

Final mark

$\frac{1}{16}$ $\left(\frac{225}{3600}\right)$

(3)

Sally is going to throw the same dice *n* times and record the number it lands on each time.
She will use her results to work out a more reliable estimate for the probability in part (*a*).

(b) What can you say about the value of *n*?

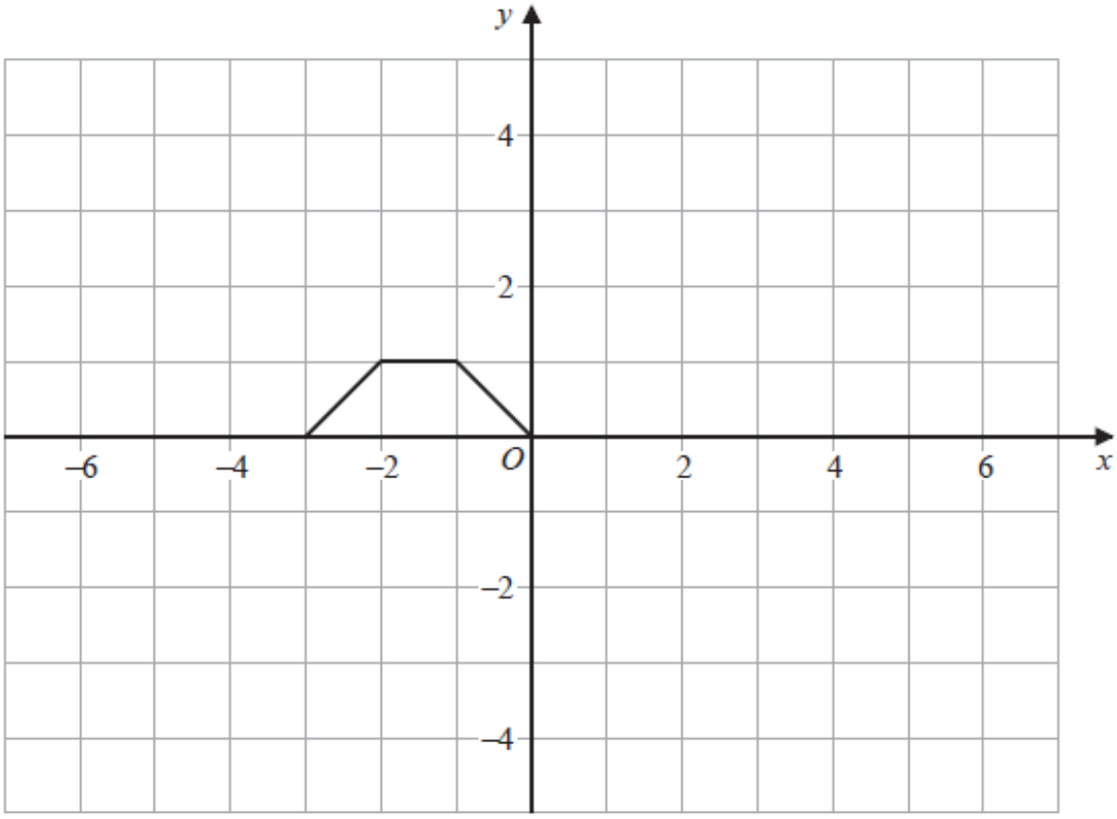
The greater the value of n, the more reliable her results will be so n should be greater than 60

1 mark

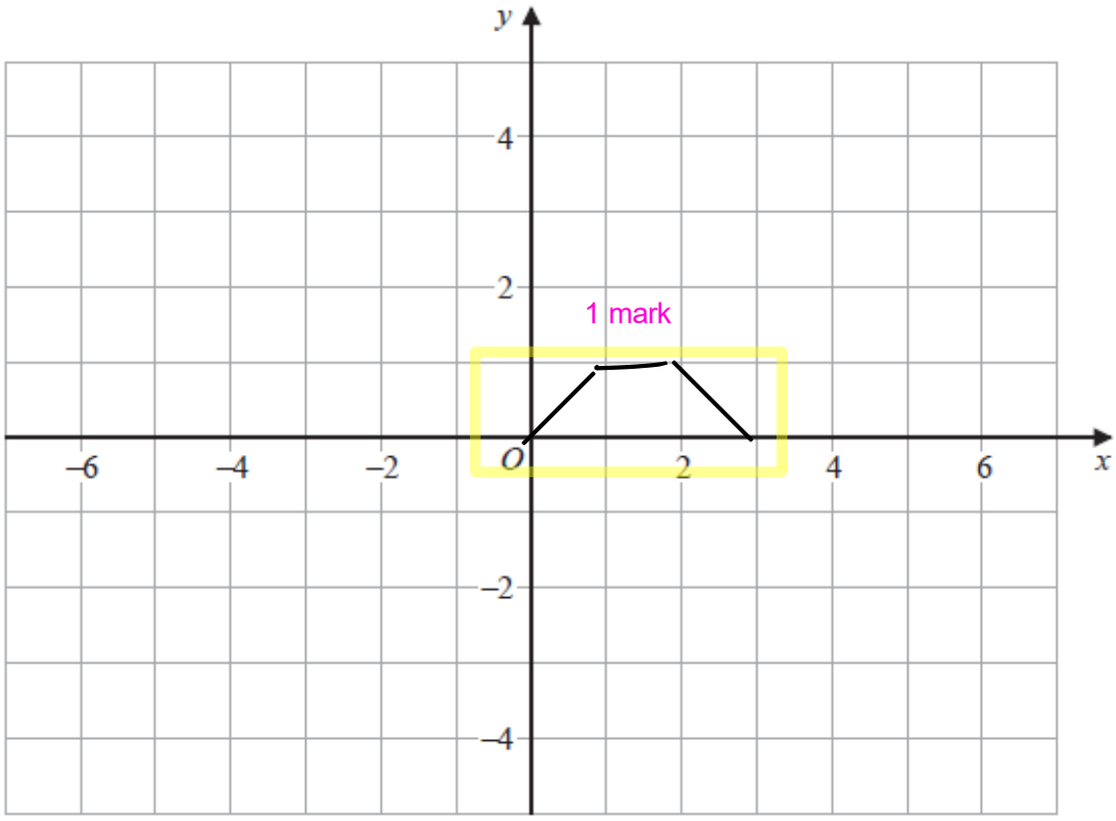
(1)

(Total for Question 2 is 4 marks)

3 Here is the graph of $y = f(x)$

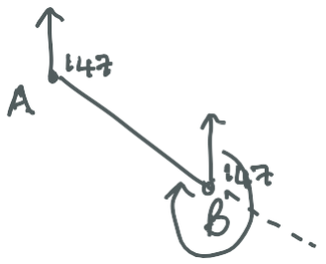


On the grid below, draw the graph of $y = f(-x)$



(Total for Question 3 is 1 mark)

***4** The bearing of port *B* from port *A* is 147°
Work out the bearing of port *A* from port *B*.



1 mark

147 + 180
= 327

Final mark

327

..... °

(Total for Question 4 is 2 marks)

---

5 $2a : 5c = 6 : 25$
$4b : 7c = 20 : 21$

Show that $a + b : b + c = 17 : 20$

2a : 5c
6 : 25

4b : 7c
20 : 21

a : c
3 : 5
9 : 15
×3

1 mark

b : c
5 : 3
25 15
×5

a : b : c

1 mark

9 : 25 : 15

Final mark

a + b = 34 b + c = 40

34 : 40

= 17 : 20 as required.

6 Write $\dfrac{14}{3x-21} + \left[(x+4) \div \dfrac{2x^2-6x-56}{2x+3}\right]$ in the form $\dfrac{ax+b}{cx+d}$ where a, b, c and d are integers.

$2x^2 - 6x - 56 = (x+4)(2x-14)$ 1 mark

$\dfrac{14}{3x-21} + (x+4) \times \dfrac{2x+3}{2x^2-6x-56}$ 1 mark

$= \dfrac{14}{3x-21} + \dfrac{(x+4)(2x+3)}{(x+4)(2x-14)}$

$= \dfrac{14(2x-14) + (2x+3)(3x-21)}{(3x-21)(2x-14)}$ 1 mark

$= \dfrac{28x - 196 + 6x^2 - 42x + 9x - 63}{(3x-21)2(x-7)}$

$= \dfrac{6x^2 - 5x - 259}{2(3x-21)(x-7)}$

$= \dfrac{(6x+37)(x-7)}{2(3x-21)(x-7)}$

$= \dfrac{6x+37}{6x-42}$ Final mark

$6x + 37$
$6x - 42$

.....

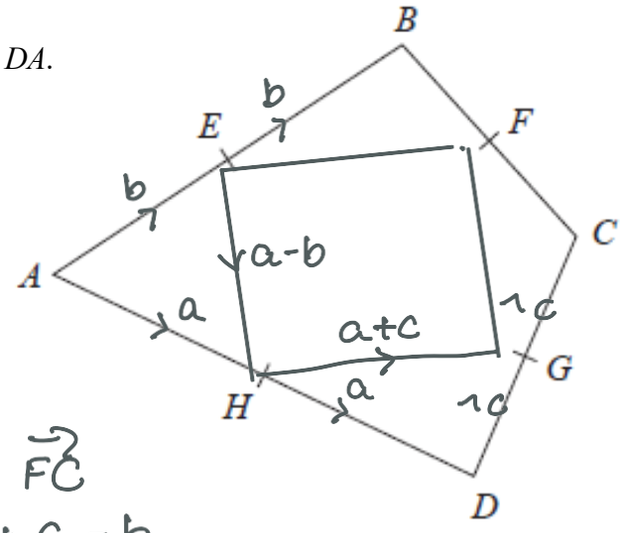
(Total for Question 6 is 4 marks)

7 $ABCD$ is a quadrilateral.

E, F, G and H are the midpoints of AB, BC, CD and DA.

$\overrightarrow{AH} = \mathbf{a}$ $\overrightarrow{AE} = \mathbf{b}$ $\overrightarrow{DG} = \mathbf{c}$

Prove, using vectors, that *EFGH* is a parallelogram.



1 mark

$\overrightarrow{BC} = 2a + 2c - 2b$ so $\overrightarrow{BF} = \overrightarrow{FC}$

$= a + c - b$

$\overrightarrow{EH} = a - b$ ← 1 mark for either of these → $\overrightarrow{HG} = a + c$

$\overrightarrow{FG} = \overrightarrow{FC} + \overrightarrow{CG}$

$= a + c - b - c$

$= a - b$

← 1 mark for either of these →

$\overrightarrow{EF} = \overrightarrow{EB} + \overrightarrow{BF}$

$= b + a + c - b$

$= a + c$

so $\overrightarrow{EH} = \overrightarrow{FG}$ so $\overrightarrow{HG} = \overrightarrow{EF}$

Final mark

$\therefore$ EFGH is a parallelogram

(Total for Question 7 is 4 marks)

---

8 Show that the equation $x^3 + 2x - 6 = 0$ has a solution between $x = 1$ and $x = 2$

$x = 1$ $1^3 + 2 \times 1 - 6 = 1 + 2 - 6 = -3$

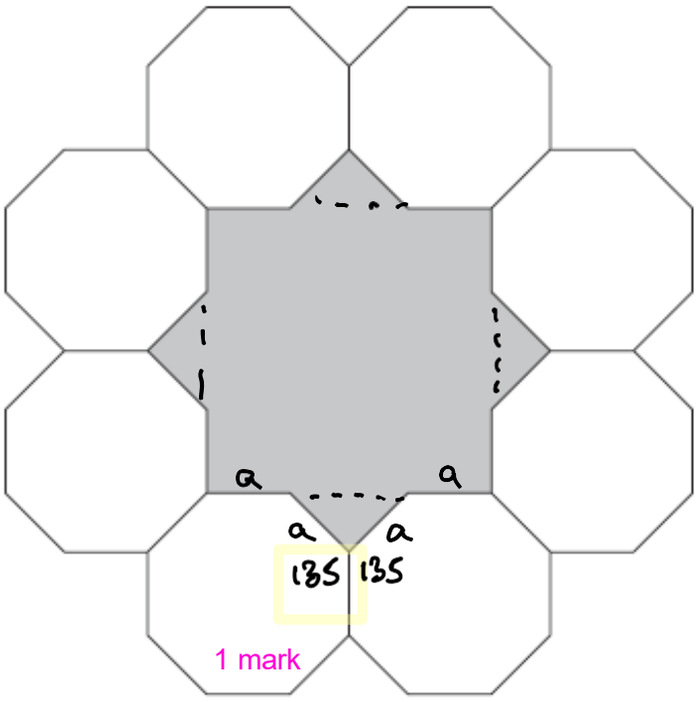
1 mark for either of these

$x = 2$ $2^3 + 2 \times 2 - 6 = 8 + 4 - 6 = 6$

Final mark

since there is a change in sign a solution must lie between $x = 1$ and $x = 2$

9 The diagram shows 8 identical regular octagons joined to enclose a shaded shape.



1 mark

(1) $1080 \div 8 = 135$

$360 - 270 = 90$

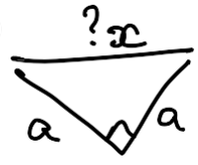
so right angle △

Each octagon has sides of length a.
Find, in terms of a, an expression for the area of the shaded shape.

Give your answer in the form $p\left(2+\sqrt{2}\right)a^2$ where p is an integer.

You must show all your working.

(2)



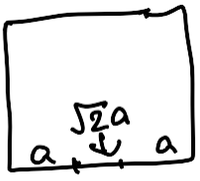
$x^2 = 2a^2$

$x = \sqrt{2}a$

1 mark

area of $\frac{1}{2}$ △ $= \frac{1}{2} a \times a = \frac{1}{2}a^2$

4 OF THESE ! $= 2a^2$



area of square

$= (2a + \sqrt{2}a)(2a + \sqrt{2}a)$

$= 4a^2 + 4\sqrt{2}a^2 + 2a^2$

$= 6a^2 + 4\sqrt{2}a^2$

1 mark

Total $= 6a^2 + 4\sqrt{2}a^2 + 2a^2 = 8a^2 + 4\sqrt{2}a^2$

Final mark $= 4(2+\sqrt{2})a^2$ as required

(Total for Question 9 is 5 marks)

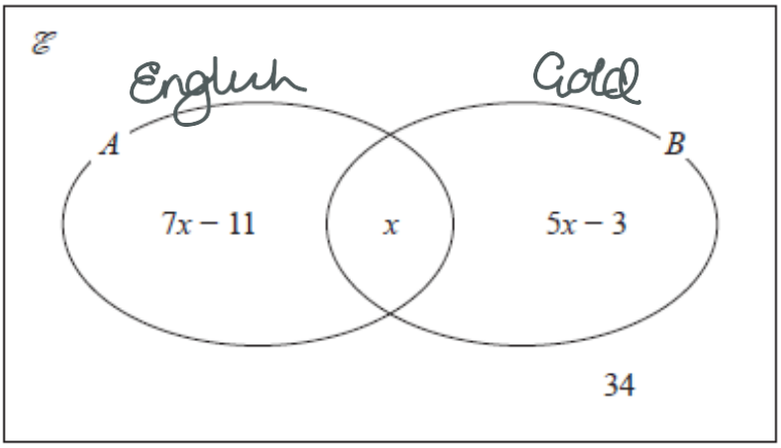
10 Vicky has a collection of medals.

The Venn diagram gives information about the number of medals in her collection where

$\mathscr{E}$ = {all medals}

A = {English medals}

B = {gold medals}



Vicky is going to take at random a medal from her collection.

Given that the medal is gold, the probability that the medal is English is $\frac{2}{11}$

Work out the number of medals in Vicky's collection.

$\mathscr{E} = 7x - 11 + x + 5x - 3 + 34$

$= 13x + 20$

Given that the medal is gold $x + 5x - 3 = 6x - 3$

$\frac{x}{6x-3} = \frac{2}{11}$ 1 mark

$11x = 2(6x - 3)$

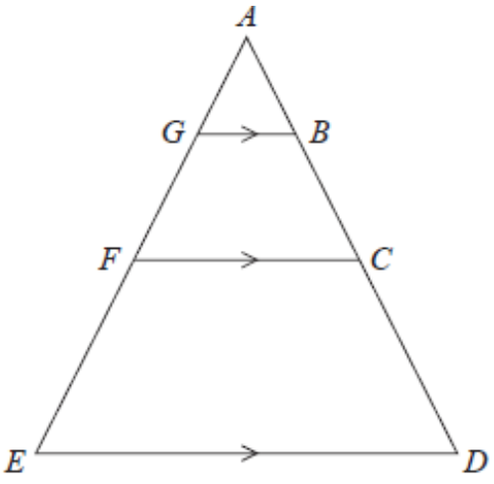
$11x = 12x - 6$ 1 mark

$x = 6$ so medals $= 13 \times 6 + 20$ 1 mark

98 Final mark

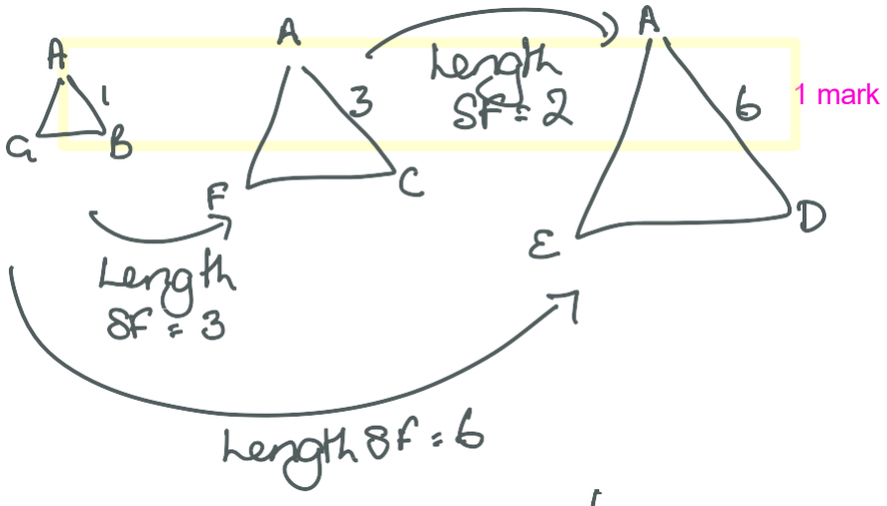
.....

11 Here are three similar triangles, *ABG*, *ACF* and *ADE*.



ABCD and *AGFE* are straight lines.
AB : *BC* : *CD* = 1 : 2 : 3
Show that

area of *ABG* : area of *BCFG* : area of *CDEF* = 1 : 8 : 27



1 mark

BCFG = area $\triangle ACF$ − $\triangle ABG$

SF = $3^2 = 9$

= 9 ABG − ABG

= 8 ABG

CDEF = $\triangle ADE$ − $\triangle ACF$

= 36 ABG − 9 ABG

= 27 ABG

∴ ratio of area 1 : 8 : 27

Final mark

(Total for Question 11 is 3 marks)

12 There are only blue pens and red pens in a box.

The number of blue pens is four times the number of red pens. Rita takes at random one pen from the box.

She records the colour of the pen and then replaces it in the box. Rita does this n times, where $n \geq 2$

Write down an expression, in terms of n, for the probability that Rita gets a blue pen at least once and a red pen at least once.

B
4R

R
R

$P(B) = \frac{4R}{5R} = \frac{4}{5}$

$P(R) = \frac{R}{5R} = \frac{1}{5}$

$P(\text{all B}) = \left(\frac{4}{5}\right)^n$

$P(\text{all R}) = \left(\frac{1}{5}\right)^n$

1 mark for either of these

Final mark

$1 - \left(\frac{4}{5}\right)^n - \left(\frac{1}{5}\right)^n$

.....

Paper 3 Solutions

Answer all questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (*a*) Factorise $a^2 - b^2$

$(a-b)(a+b)$

1 mark

$(a-b)(a+b)$

(1)

(*b*) Show that $2^{40} - 1$ is the product of two consecutive odd numbers.

$a^2 - b^2$ so $(2^{20})^2 - 1^2$

$= (2^{20}-1)(2^{20}+1)$ 1 mark

2^{20} = even $2^{20}-1$ = odd

so $2^{20}+1$ is next consecutive odd number.

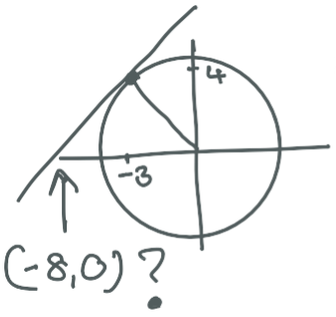
(2)

2 A circle has equation $x^2 + y^2 = 25$

The point *P* with coordinates (–3, 4) lies on the circle.

Alex says that the tangent to the circle at *P* crosses the *x*-axis at the point (–8, 0) Is Alex correct?

You must show how you get your answer.



$x^2+y^2 = 25$ so radius = 5

gradient of radius $= -\frac{4}{3}$ 1 mark

gradient of tangent $= \frac{3}{4}$ 1 mark

equation of tangent $y = \frac{3}{4}x + c$

(-3, 4)
x y

$4 = \frac{3}{4} \times -3 + c$

$c = 4 + \frac{9}{4} = 6.25$

so $y = 0.75x + 6.25$ 1 mark

when $y = 0$

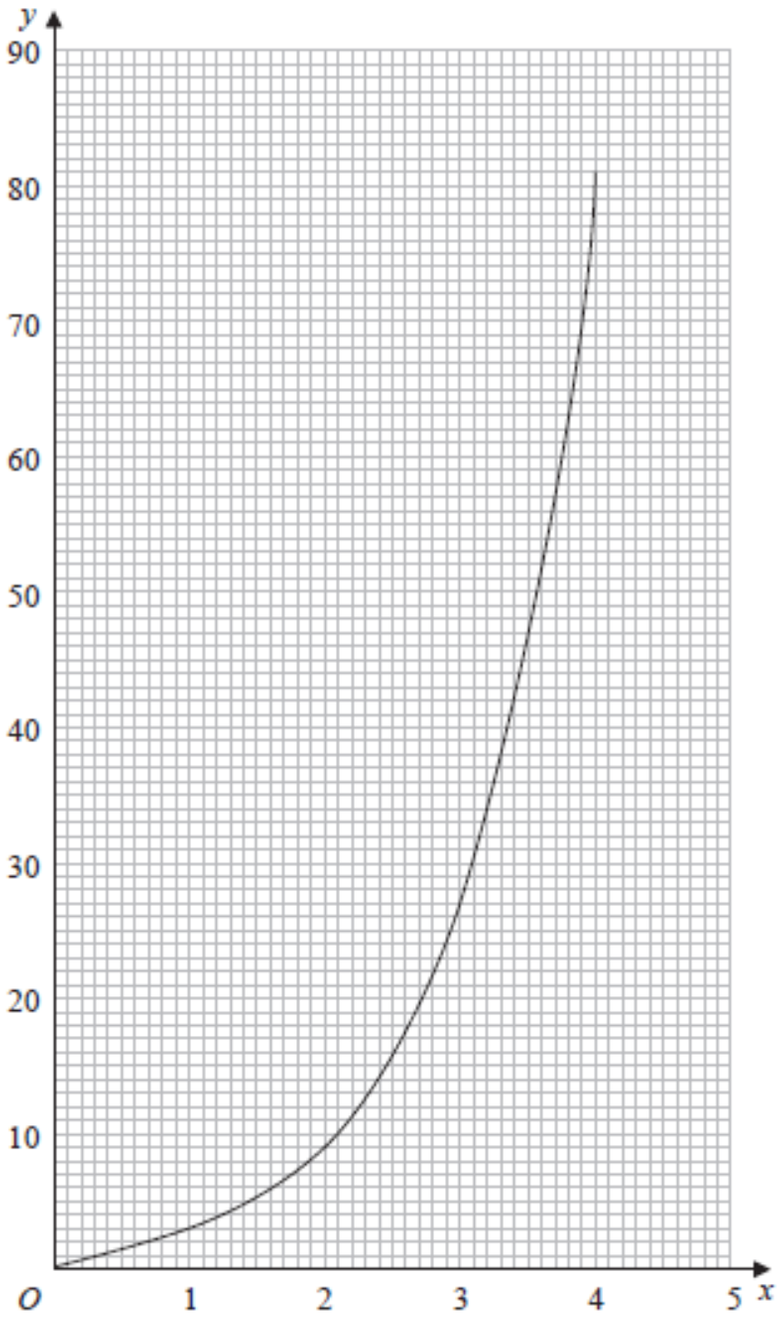
$0.75x = -6.25$

$x = -\frac{6.25}{0.75} = -8\frac{1}{3}$

so Alex is not correct.

3 Sana needs to draw the graph of $y = 3^x$ for $0 \leq x \leq 4$

She draws the graph shown on the grid.

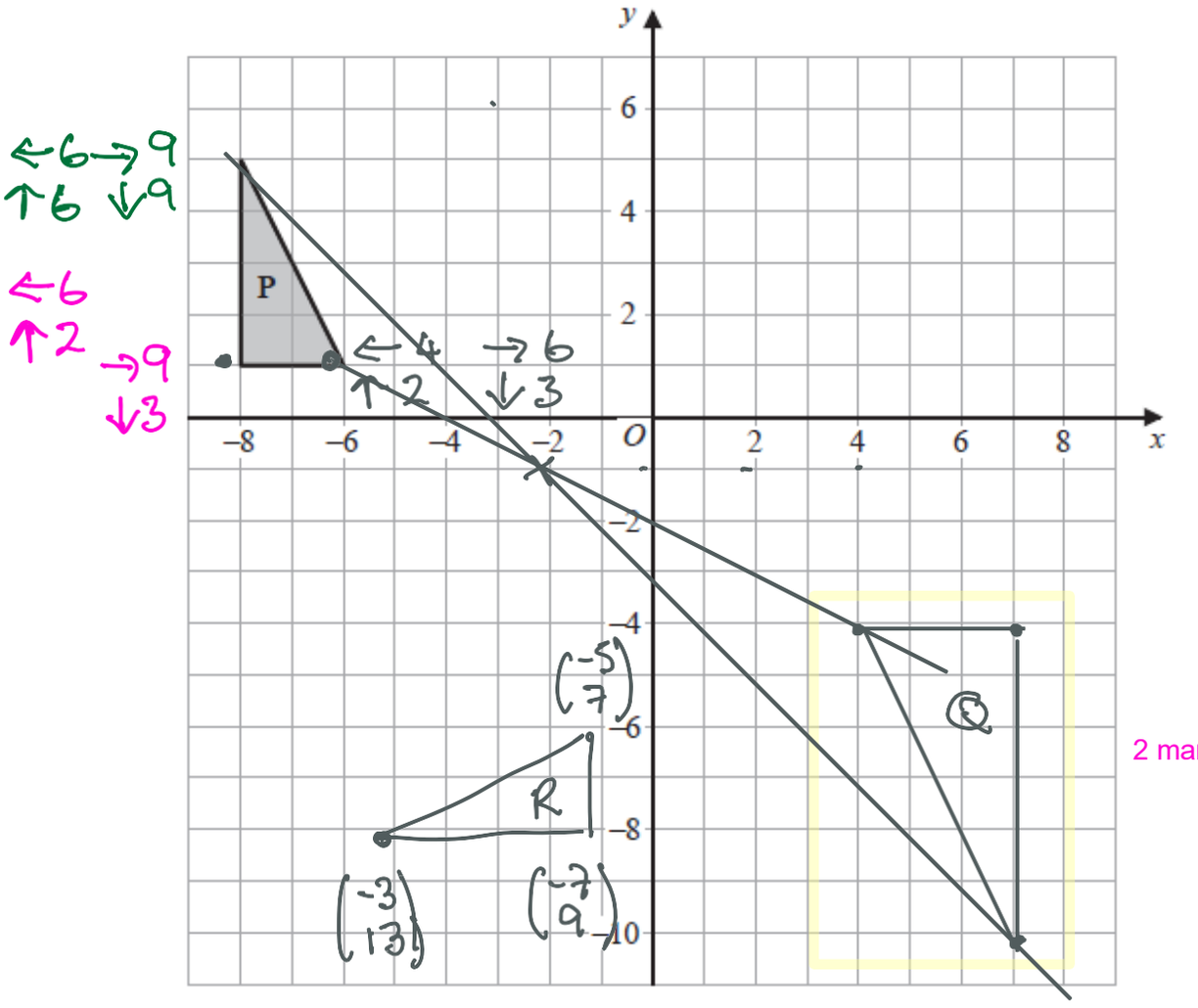


Write down one thing Sana has done wrong.

1 mark

When $x = 0$ $y = 3^0 = 1$ and not zero.
so the line should go through (0,1) not (0,0)

4



2 marks

(a) Enlarge triangle **P** by scale factor $-1\frac{1}{2}$ with centre of enlargement (–2, –1)

Label your image **Q**.

(2)

Triangle **P** is transformed by a combined transformation of a rotation of 90° anticlockwise about the origin followed by a translation to give triangle **R**.
Exactly one vertex of triangle **P** is invariant under the combined transformation.

1 mark for any one of these

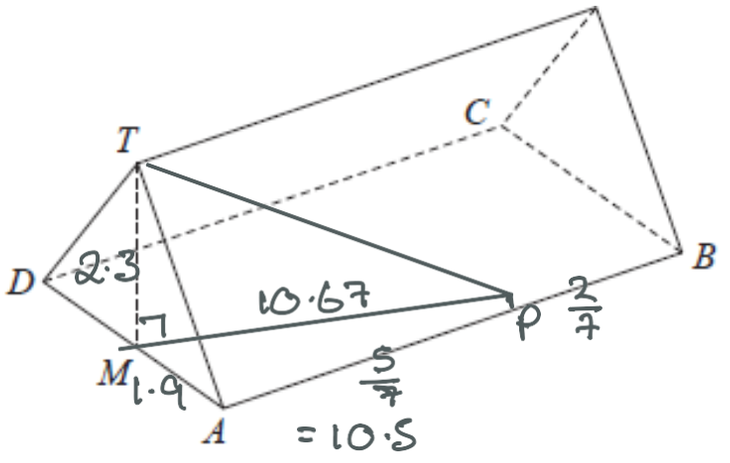
(b) Find one possible column vector for the translation.

3 solutions possible

$\begin{pmatrix}-3\\13\end{pmatrix}$ $\begin{pmatrix}-7\\9\end{pmatrix}$ or $\begin{pmatrix}-5\\7\end{pmatrix}$

(1)

5 The diagram shows a triangular prism with a horizontal rectangular base *ABCD*.



M is the midpoint of *AD*.

The vertex *T* of the prism is vertically above *M*.

$AB = 14.7$ cm $\quad BC = 3.8$ cm $\quad MT = 2.3$ cm

P is the point on *AB* such that

$$AP : PB = 5 : 2$$

Calculate the size of the angle between *TP* and the base *ABCD* of the prism.

Give your answer correct to 1 decimal place.

$AP = \frac{5}{7} \times 14.7 = 10.5$ 1 mark

$MP^2 = \sqrt{10.5^2 + 1.9^2}$ 1 mark

$= \sqrt{113.86}$

$= 10.67$

$\tan TPM = \frac{2.3}{10.67}$ 1 mark

$TPM = \tan^{-1} \frac{2.3}{10.67}$

$= 12.1638..$

Final mark

12·2

..... °

6 An expression for the nth term of the sequence of triangular numbers is $\frac{n(n+1)}{2}$

Prove that the sum of any two consecutive triangular numbers is a square number.

$\frac{n(n+1)}{2} + \frac{(n+1)(n+1+1)}{2}$ 1 mark

$= \frac{n^2+n}{2} + \frac{(n+1)(n+2)}{2}$ 1 mark

$= \frac{n^2+n+n^2+3n+2}{2} = \frac{2n^2+4n+2}{2}$

$= n^2+2n+1 = (n+1)(n+1) = (n+1)^2$ Final mark

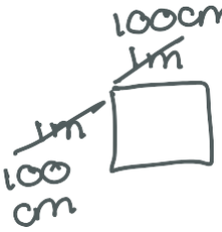
which is a square number.

---

7 The floor plan of a house is drawn using a scale of 1 : 50
On the plan, a room in the house has a floor area of 48 cm^2
Work out the real area of the floor of this room.
Give your answer in m^2

length SF = 50 1 mark

so area SF = $50^2 = 2500$



$48 \times 2500 = 120\,000 cm^2$

$1m^2 = 10{,}000 cm^2$

1 mark

so $120000 cm^2 \div 10000 cm^2$

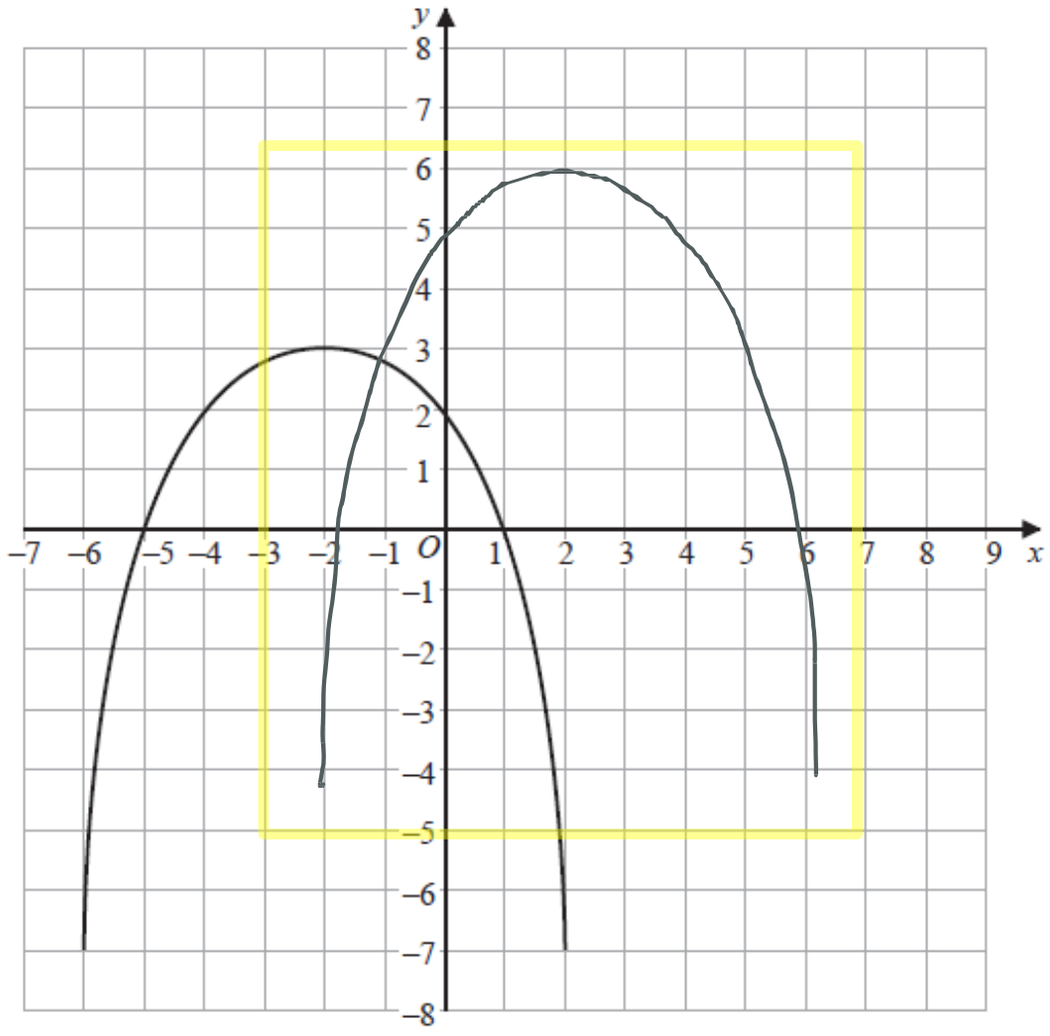
$= 12$

12 Final mark

. ..... m^2

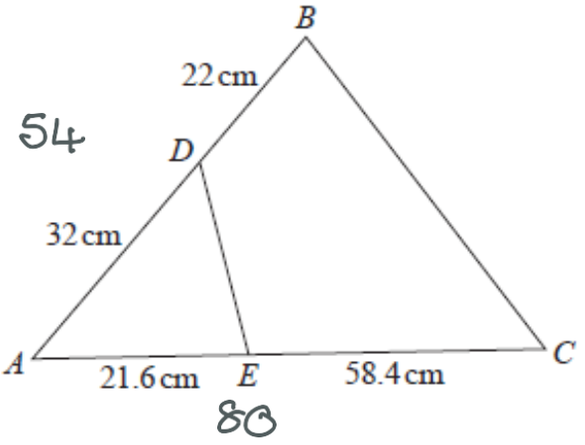
8 The graph of $y = f(x)$ is shown on the grid.

2 marks for correct sketch



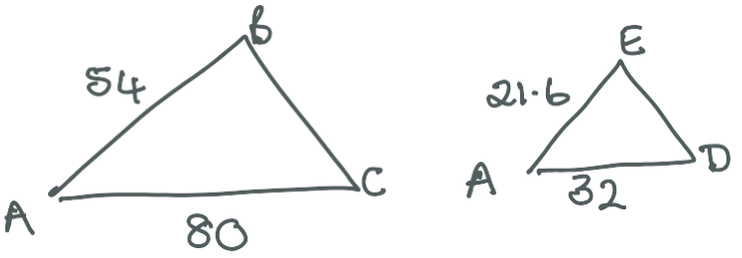
On the grid, sketch the graph of $y = f(-x) + 3$

9 The diagram shows triangle *ABC* and triangle *AED*.



Show that triangle *ABC* and triangle *AED* are similar.

LOOK at the order of the letters



$80 \div 32 = 2.5$

$54 \div 21.6 = 2.5$

1 mark

Both triangle share angle A and the sides have a common scale factor.

10 There is a total of y counters in a box.

There are x pink counters and 5 blue counters in the box.

The rest of the counters are green.

$$x : y = 1 : 3$$

1 mark

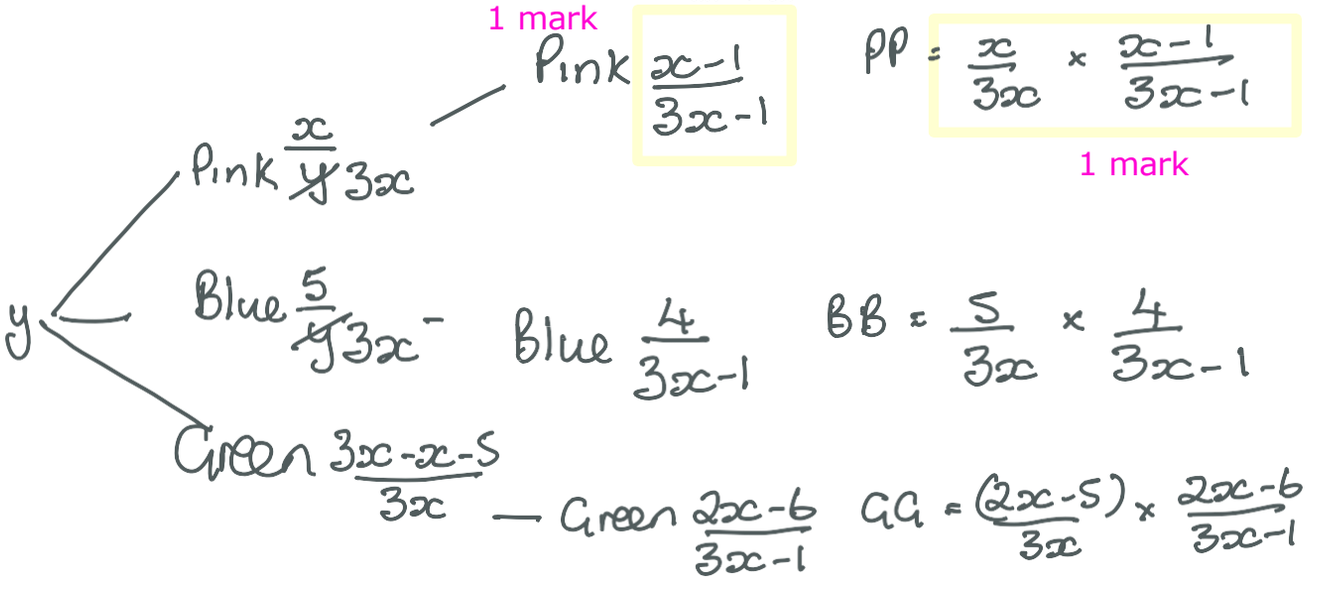
$\frac{x}{y} = \frac{1}{3}$ $\quad y = 3x$

Freda takes at random two counters from the box.

Find, in terms of x, an expression for the probability that Freda takes two counters of the same colour.

Give your answer as a fraction in the form $\frac{ax^2+bx+c}{dx^2+ex}$ where a, b, c, d and e are integers.

1 mark



$$P(\text{same}) = \frac{x^2-x}{3x(3x-1)} + \frac{20}{3x(3x-1)} + \frac{(2x-5)(2x-6)}{3x(3x-1)}$$

1 mark

$$= \frac{x^2 - x + 20 + 4x^2 - 12x - 10x + 30}{3x}$$

$9x^2 -$

Final mark

$$= \frac{5x^2 - 23x + 50}{9x^2 - 3x}$$

$\frac{5x^2 - 23x + 50}{9x^2 - 3x}$

11 Ebony makes some bracelets to sell.

The materials to make all the bracelets cost £190, correct to the nearest £5

Ebony sells all the bracelets for a total of £875, correct to the nearest £5

The total time taken to make and sell all these bracelets was 72 hours, correct to the nearest hour.

Ebony uses this method to calculate her hourly rate of pay

$$\text{Hourly rate of pay} = \frac{\text{total selling price} - \text{total cost of materials}}{\text{total time taken}}$$

The minimum hourly rate of pay for someone of Ebony's age is £8.20

By considering bounds, determine if Ebony's hourly rate of pay was definitely more than £8.20

You must show all your working.

materials 190 nearest £5 → 192.50, 187.50

sells 875 nearest £5 → 877.50, 872.50

time 72 hours nearest hour → 72.5, 71.5

1 mark for any one of these bounds

Looking for LB of hourly rate of pay.

$$= \frac{872.50 - 192.50}{72.50}$$

1 mark

1 mark

$= 9.3751\ldots$ so £9.38

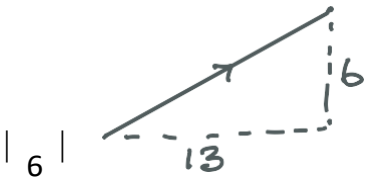
Yes it is definitely more than £8.20.

12 Given that the vector

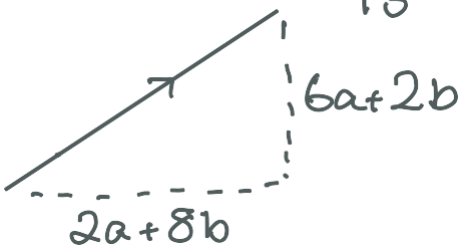
$a\binom{2}{\ }+b\binom{8}{\ }$ $|_6|$ $|_2|$

is parallel to the vector $\begin{pmatrix} 13 \\ 6 \end{pmatrix}$

find an expression for b in terms of a.



gradient = $\frac{b}{13}$



1 mark

① $6a + 2b = 6$ × 4

② $2a + 8b = 13$

③ $24a + 8b = 24$

③ − ② $22a = 11$

$a = 0.5$ 1 mark

sub in ② $1 + 8b = 13$

$8b = 12$

$3a$

$b = \frac{3}{2} = 1.5$

$a = 0.5$ $b = 1.5$

so $b =$

Final mark

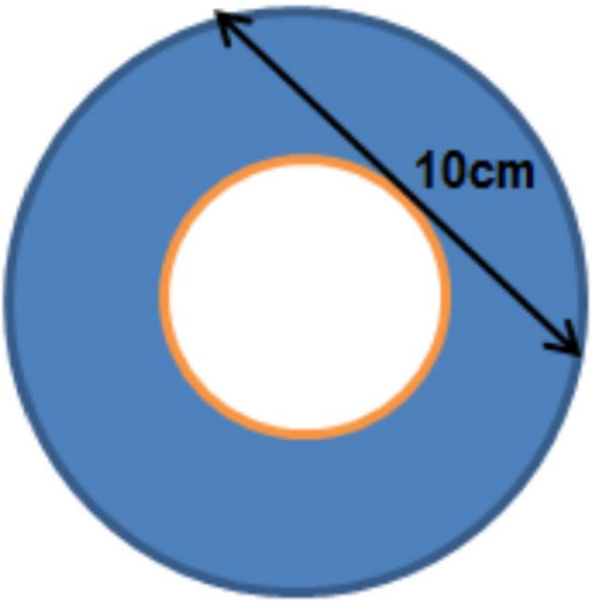
$b = 3a$

.....

Problems 1,2 & 3

Circle areas

Can you work out the shaded area in the diagram (the line shown just touches the smaller circle)?



Find the value of

$$\frac{99}{100} \times \frac{80}{81} \times \frac{63}{64} \times \frac{48}{49} \times \frac{35}{36} \times \frac{24}{25} \times \frac{15}{16} \times \frac{8}{9} \times \frac{3}{4}.$$

Write your answer in the form $\frac{a}{b}$, where a and b are positive integers with no common factors other than 1.

A point E lies outside the rectangle $ABCD$ such that CBE is an equilateral triangle. The area of the pentagon $ABECD$ is five times the area of the triangle CBE.

What is the ratio of the lengths $AB : AD$?

Write your answer in the form $a : 1$.

Problems 4,5 & 6

A sequence is defined as follows:

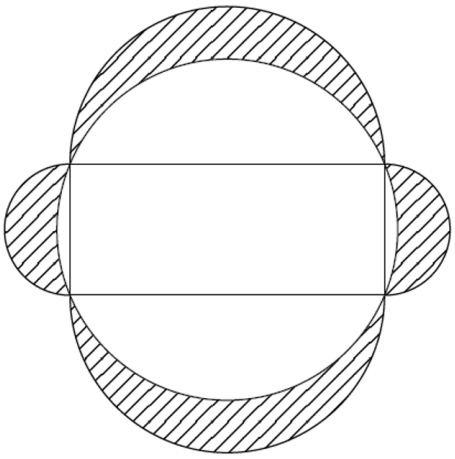
$u_1 = 123$.

For $n \geq 1$, define u_{n+1} = the sum of the squares of the digits of u_n.

For example, $u_2 = 1^2 + 2^2 + 3^2 = 14$, $u_3 = 1^2 + 4^2 = 17$.

What is the value of u_{100}?

Four semicircles are drawn on the sides of a rectangle with width 10 cm and length 24 cm. A circle is drawn that passes through the four vertices of the rectangle.



What is the value, in cm^2, of the shaded area?

Alfred, Brenda, Colin, David and Erica have to sit on a row of five chairs. Alfred does not want to sit next to Brenda. David does not want to sit next to Erica.

In how many ways can these five people arrange themselves and ensure the above conditions are met?

Problems 7 & 8

(a) Which positive integer in the range from 1 to 250 has more different prime divisors than any other integer in this range?

[3 marks]

(b) When $n = 5$ the product $n(n + 1)(n + 2)$ can be written as the product of four distinct primes. Indeed, when $n = 5$

$$n(n + 1)(n + 2) = 5 \times 6 \times 7 = 2 \times 3 \times 5 \times 7.$$

What is the least positive integer n such that $n(n + 1)(n + 2)$ can be written as a product of *five* distinct primes? [3 marks]

Find the value of

$$\left(\left(2^{\frac{3}{4}}+1\right)^2+\left(2^{\frac{3}{4}}-1\right)^2\right)\left(\left(2^{\frac{3}{4}}+1\right)^2+\left(2^{\frac{3}{4}}-1\right)^2-2^2\right).$$