



DIXONS
SIXTH FORM
ACADEMY

**SUMMER
WORK
2022**

A Level
Mathematics

STUDENT NAME:



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About the Summer Work

This booklet contains a number of tasks that students are expected to complete to a good standard in order to be enrolled in this subject.

Please complete these tasks on A4 paper and bring them with you to your first maths lesson.

The work handed in should be:

- written in black or blue ink on A4 lined paper with pencil for graph drawing
- exercises labelled and question numbers included
- work should be self-marked and corrected for errors
- have student's full name on each sheet
- multiple sheets should be stapled together

This booklet also contains significant additional information. We would encourage you to complete all the tasks including the optional ones to fully prepare for Sixth Form study.

Calculators may be used to support your answer but working must be shown. Video links are included with each exercise if you require further examples that are worked through, but all work is predominantly revision from GCSE.



Welcome to Mathematics

Subject outline

A-level maths is the most popular A-level taken by students who go on to university. The subject sharpens many key skills, including the ability to get to grips with problems, something that lies at the centre of many fields. Students who study maths at A-Level relish a challenge and enjoy investigating different processes. Successful students understand the power of practise in mathematics and invest their study time in completing exercises and past exam questions frequently so as to become familiar with a range of different contexts. Outside of lesson time, students need to use their independent study time to practice maths and check through solutions from a range of resources including online retrieval practise exercises, textbook procedural practise and past exam papers.

Mathematics has always been a highly valued A-Level by Universities and employers due to its complex content and the demands of the course. Mathematics opens pathways for students to a wide range of courses that require students to be highly numerate and strong logical thinkers. In our technology focussed society, mathematics students can often show innovation and creativity in approaching a challenge and working to find a solution, traits which are essential in the modern-day work force.

Students will study the Edexcel Specification for A Level Mathematics, with three 2-hour papers at the end of year 2.

Paper 1&2 – Pure Mathematics (concepts such as algebra, coordinate geometry, graphs and calculus)

Paper 3 – Mechanics and Statistics (including forces, kinematics and projectiles for mechanics and probability distributions, data handling, hypothesis testing in statistics)



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Careers & Higher Education

A Level Mathematics is a facilitating subject, meaning that it is a highly respected A Level qualification that is listed as essential on some university courses. If you are interested in studying engineering, economics, mathematics, physics, statistics, actuarial science or computer science most universities require you to complete A Level Mathematics. If you are interested in other routes such as biochemistry, dentistry, business studies, geography or accounting some universities may list mathematics as a useful subject but not essential.

Maths careers are some of the most highly paid careers available. Research shows that on average A Level maths students earn 11% more than other students during their lifetime. Many believe that taking maths at university has limited fields since it doesn't move straight into a vocation. However, this is certainly not the case. Students who continue maths at university can move into various careers, from graduate roles within the finance industry to working in a graduate role within the civil service. Engineering has many different degree routes and courses and is one of the most popular areas that students choose to work in after university.

An example of a highly mathematical career is an actuary. An actuary works in a business analysing risk, often within the financial sector. Actuaries use mathematical modelling techniques and statistical concepts to determine probability and assess risks, for example, analysing pension scheme liabilities to price commercial insurance. Due to the challenging nature of the exams required to become a qualified actuary, the salary is usually very competitive.

Links to key information:

dixons6a.com/uploads/files/Maths.pdf

qualifications.pearson.com/en/qualifications/edexcel-a-levels/mathematics-2017.html



Summer work tasks

Task 1 – practise exercises

There are 8 practise tasks to complete, with examples and answers, that are all revision of key topics from GCSE. These topics are essential to the study of A Level Mathematics and students need to ensure they fully understand each concept. **Students will be assessed in the first lesson on these topics** to ensure they are starting the course with a grounded understanding of algebra from GCSE. Questions should be completed on paper and should be clearly self-marked. All solutions are provided. The exercises are provided at the end of this pack.

Task 2 – online assessment

We have set an online assessment that you access by creating an account with Dr Frost Maths. To complete please follow the instructions below. Your score from this will be recorded and sent to your new maths teacher so they will be able to assess your current understanding. **This should be done after the practise exercises are completed.** There are a total of 50 questions, but you do not need to complete all at once. Your progress will be saved and you can return to it at another time.



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To complete this please follow the instructions below.

Follow this link: drfrostmaths.com/register.php?cid=203395&passcode=66401

Step 1: do not click that you have a school account. Please make a **new account using your personal email address** in the part of the form you see here:

...or fill in this information.

School: **Dixons Free Sixth Form**

First Name:

Surname:

~~School~~ Email:

Please use a personal email here – we won't see this email, but your Dr Frost Account will be linked to it.

~~If you have a school email address, please use it. You are allowed to use a personal email address, and teachers will not be able to see it. We use your email address only to automatically contact you when homework is set or is late.~~

Set a Password:

Confirm Password:

I'm Done

Step 2: After you should see this message:

You've successfully registered. Just click the link in the email we've sent you and you're good to go.

please click the link in order to sign up.



SUMMER WORK 2022

Step 3: Now wait... The task will be assigned to your email address after your account has been made. You will receive another email when the task has been set. If you have not received anything within 24 hours please contact dsmith@dixons6a.com with your full name.

Once set you will see this on your dashboard when you log in – remember this will not appear straight away but you will receive an email when it is set.

The screenshot shows a dashboard with several sections:

- Practice:** Key Skills: 0/924, Topic Medals: 0/588. Includes a donut chart and a 'Start Practising' button.
- Latest Homework:** Set by Ms Smith, "Enrolment Summary Work 2022". Includes a 'Review All' button. This section is circled in red.
- A Level (Yrs1-2):** Work through a course, with a mix of question practice, worked example videos and downloadable resources. Includes a 'Continue this Course' button.
- Progress:** See a timeline of your activity and progress by topic. Includes a 'Go' button.
- Trophies: 0/37:** You earn trophies for achievements related to answering questions. Includes a 'View' button.
- Clean Up:** Do a quick task consisting of 4 questions you've recently got wrong. Includes a 'Go' button.

Complete the latest homework section circled above.



Task 3 – further practise (optional)

Use the GCSE key skills on Dr Frost Maths to complete extra practise. Explore any of the algebraic manipulation topics that appeared in the online assessment. Some students find the start of term very demanding, with a lot of changes to get used to. Students without a secure understanding of GCSE skills find it particularly so. Use your spare time over the summer wisely to put yourself in as good a position as possible for the start of your A level course.

The screenshot displays a dashboard with several interactive cards:

- Practice**: Shows 'Key Skills: 0/924' and 'Topic Medals: 0/588' with a donut chart. A red circle highlights this card. Below the chart, it says 'Build knowledge using Key Skills, accompanied by worked example videos, before broader practice on exam questions.' and has a 'Start Practising' button.
- Latest Homework**: Shows 'Set by Ms Smith' and '"Enrolment Summary Work 2022"' with a 'Review All' button.
- A Level (Yrs1-2)**: A dark card with a 'Continue this Course' button.
- Progress**: Says 'See a timeline of your activity and progress by topic.' with a 'Go' button.
- Trophies: 0/37**: Says 'You earn trophies for achievements related to answering questions.' with a 'View' button.
- Clean Up**: Says 'Do a quick task consisting of 4 questions you've recently got wrong.' with a 'Go' button.



Reading list

Suggested reading:

[The Codebook](#) by *Simon Singh*

[Infinity: The Quest to Think the Unthinkable](#) by *Brian Clegg*

[The Man who knew Infinity](#) by *Robert Kanigel*

Suggested viewing:

bbc.co.uk/iplayer/episode/b0074rxx/horizon-19951996-fermats-last-theorem

Exercise 1: Simplifying indices

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the n th root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Example 1 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$
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Example 2 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	<ol style="list-style-type: none"> 1 Use the rule $a^m \times a^n = a^{m+n}$ 2 Use the rule $\frac{a^m}{a^n} = a^{m-n}$
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Example 3 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged
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Example 4 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\begin{aligned} \frac{4}{\sqrt{x}} &= \frac{4}{x^{\frac{1}{2}}} \\ &= 4x^{-\frac{1}{2}} \end{aligned}$	<ol style="list-style-type: none"> 1 Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ 2 Use the rule $\frac{1}{a^m} = a^{-m}$
---	---

Practice questions

1 Simplify.

a $\frac{3x^2 \times x^3}{2x^2}$

b $\frac{10x^5}{2x^2 \times x}$

c $\frac{3x \times 2x^3}{2x^3}$

d $\frac{7x^3y^2}{14x^5y}$

e $\frac{y^2}{y^{\frac{1}{2}} \times y}$

f $\frac{c^{\frac{1}{2}}}{c^2 \times c^{\frac{3}{2}}}$

g $\frac{(2x^2)^3}{4x^0}$

h $\frac{x^{\frac{1}{2}} \times x^{\frac{3}{2}}}{x^{-2} \times x^3}$

Watch out!

Remember that any value raised to the power of zero is 1. This is the rule $a^0 = 1$.

2 Write the following as a single power of x .

a $\frac{1}{x}$

b $\frac{1}{x^7}$

c $\sqrt[4]{x}$

d $\sqrt[5]{x^2}$

e $\frac{1}{\sqrt[3]{x}}$

f $\frac{1}{\sqrt[3]{x^2}}$

3 Write the following without negative or fractional powers.

a x^{-3}

b x^0

c $x^{\frac{1}{5}}$

d $x^{\frac{2}{5}}$

e $x^{-\frac{1}{2}}$

f $x^{\frac{3}{4}}$

Answers

1 a $\frac{3x^3}{2}$

c $3x$

e $y^{\frac{1}{2}}$

g $2x^6$

b $5x^2$

d $\frac{y}{2x^2}$

f c^{-3}

h x

2 a x^{-1}

d $x^{\frac{2}{5}}$

b x^{-7}

e $x^{-\frac{1}{3}}$

c $x^{\frac{1}{4}}$

f $x^{\frac{2}{3}}$

3 a $\frac{1}{x^3}$

d $\sqrt[5]{x^2}$

b 1

e $\frac{1}{\sqrt{x}}$

c $\sqrt[5]{x}$

f $\frac{1}{\sqrt[4]{x^3}}$

Exercise 2: Simplifying fractions

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- $a^m \times a^n = a^{m+n}$
- $\frac{a^m}{a^n} = a^{m-n}$
- $(a^m)^n = a^{mn}$
- $a^0 = 1$
- $a^{\frac{1}{n}} = \sqrt[n]{a}$ i.e. the n th root of a
- $a^{\frac{m}{n}} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$
- $a^{-m} = \frac{1}{a^m}$
- The square root of a number produces two solutions, e.g. $\sqrt{16} = \pm 4$.

Example 1 Evaluate 10^0

$10^0 = 1$	Any value raised to the power of zero is equal to 1
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Example 2 Evaluate $9^{\frac{1}{2}}$

$9^{\frac{1}{2}} = \sqrt{9}$ $= 3$	Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$
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Example 3 Evaluate $27^{\frac{2}{3}}$

$27^{\frac{2}{3}} = (\sqrt[3]{27})^2$ $= 3^2$ $= 9$	<ol style="list-style-type: none"> 1 Use the rule $a^{\frac{m}{n}} = (\sqrt[n]{a})^m$ 2 Use $\sqrt[3]{27} = 3$
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Example 4 Evaluate 4^{-2}

$4^{-2} = \frac{1}{4^2}$ $= \frac{1}{16}$	<ol style="list-style-type: none"> 1 Use the rule $a^{-m} = \frac{1}{a^m}$ 2 Use $4^2 = 16$
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Example 5 Simplify $\frac{6x^5}{2x^2}$

$\frac{6x^5}{2x^2} = 3x^3$	$6 \div 2 = 3$ and use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$
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Example 6 Simplify $\frac{x^3 \times x^5}{x^4}$

$\frac{x^3 \times x^5}{x^4} = \frac{x^{3+5}}{x^4} = \frac{x^8}{x^4}$ $= x^{8-4} = x^4$	<ol style="list-style-type: none"> 1 Use the rule $a^m \times a^n = a^{m+n}$ 2 Use the rule $\frac{a^m}{a^n} = a^{m-n}$
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Example 7 Write $\frac{1}{3x}$ as a single power of x

$\frac{1}{3x} = \frac{1}{3}x^{-1}$	Use the rule $\frac{1}{a^m} = a^{-m}$, note that the fraction $\frac{1}{3}$ remains unchanged
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Example 8 Write $\frac{4}{\sqrt{x}}$ as a single power of x

$\frac{4}{\sqrt{x}} = \frac{4}{x^{\frac{1}{2}}}$ $= 4x^{-\frac{1}{2}}$	<ol style="list-style-type: none"> 1 Use the rule $a^{\frac{1}{n}} = \sqrt[n]{a}$ 2 Use the rule $\frac{1}{a^m} = a^{-m}$
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Example 9 Simplify $\frac{x^5}{x^2}$

$\frac{x^5}{x^2} = x^3$	use the rule $\frac{a^m}{a^n} = a^{m-n}$ to give $\frac{x^5}{x^2} = x^{5-2} = x^3$
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Example 10 Simplify $6x^6 \times 3x^4$

$6x^6 \times 3x^4 = 18x^{10}$	$6 \times 3 = 18$ and then use the rule $a^m \times a^n = a^{m+n}$ to give $x^6 \times x^4 = x^{6+4} = x^{10}$
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Example 11 Simplify $(x^4)^2 \times 3x^5$

$(x^4)^2 \times 3x^5 = 3x^{13}$	$3 \times 1 = 3$ and then use the rule $(a^m)^n = a^{mn}$ following by to give $a^m \times a^n = a^{m+n}$ $(x^4)^2 \times x^5 = x^{4 \times 2} \times x^5$ $= x^8 \times x^5$ $= x^{8+5}$ $= x^{13}$
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Practice questions

1 Write as sums of powers of x .

a $\frac{x^5 + 1}{x^2}$

b $x^2 \left(x - \frac{1}{x} \right)$

c $x^{-4} \left(x^2 + \frac{1}{x^3} \right)$

d $\frac{6x^5 + 3x^4}{3x^2}$

e $\frac{5x^5 + 20x^4}{10x^2}$

f $\frac{7x^5 - 5x^4}{2x^6}$

Answers

1 a $x^3 + x^{-2}$

b $x^3 - x$

c $x^{-2} + x^{-7}$

d $2x^3 + x^2$

e $0.5x^3 + 2x^2$

f $3.5x^{-1} - 2.5x^{-2}$

Exercise 3: Expanding brackets and surds

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Key points

- A surd is the square root of a number that is not a square number, for example $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, etc.
- Surds can be used to give the exact value for an answer.
- $\sqrt{ab} = \sqrt{a} \times \sqrt{b}$
- $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$
- To rationalise the denominator means to remove the surd from the denominator of a fraction.
- To rationalise $\frac{a}{\sqrt{b}}$ you multiply the numerator and denominator by the surd \sqrt{b}
- To rationalise $\frac{a}{b + \sqrt{c}}$ you multiply the numerator and denominator by $b - \sqrt{c}$

Example 1 Simplify $(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2})$

$ \begin{aligned} &(\sqrt{7} + \sqrt{2})(\sqrt{7} - \sqrt{2}) \\ &= \sqrt{49} - \sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} - \sqrt{4} \\ &= 7 - 2 \\ &= 5 \end{aligned} $	<ol style="list-style-type: none"> Expand the brackets. A common mistake here is to write $(\sqrt{7})^2 = 49$ Collect like terms: $\begin{aligned} &-\sqrt{7}\sqrt{2} + \sqrt{2}\sqrt{7} \\ &= -\sqrt{7}\sqrt{2} + \sqrt{7}\sqrt{2} = 0 \end{aligned}$
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Practice questions

1 Expand and simplify.

a $(\sqrt{2} + \sqrt{3})(\sqrt{2} - \sqrt{3})$

b $(3 + \sqrt{3})(5 - \sqrt{12})$

c $(4 - \sqrt{5})(\sqrt{45} + 2)$

d $(5 + \sqrt{2})(6 - \sqrt{8})$

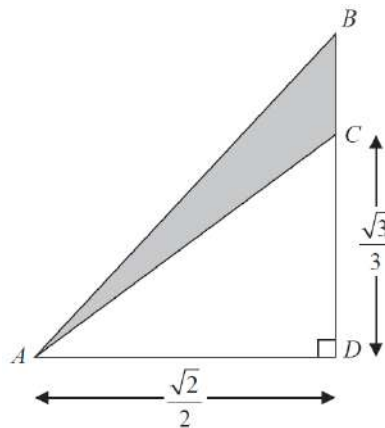
2 Expand and simplify $(\sqrt{x} + \sqrt{y})(\sqrt{x} - \sqrt{y})$

3 Work out the value of $(\sqrt{2} + \sqrt{8})^2$

4 Expand $(1 + \sqrt{2})(3 - \sqrt{2})$

Give your answer in the form $a + b\sqrt{2}$ where a and b are integers.

5 ABD is a right angled triangle.



All measurements are given in centimetres.

C is the point on BD such that $CD = \frac{\sqrt{3}}{3}$

$$AD = BD = \frac{\sqrt{2}}{2}$$

Work out the exact area, in cm^2 , of the shaded region.

6 The diagram shows a triangle DEF inside a rectangle $ABCD$.

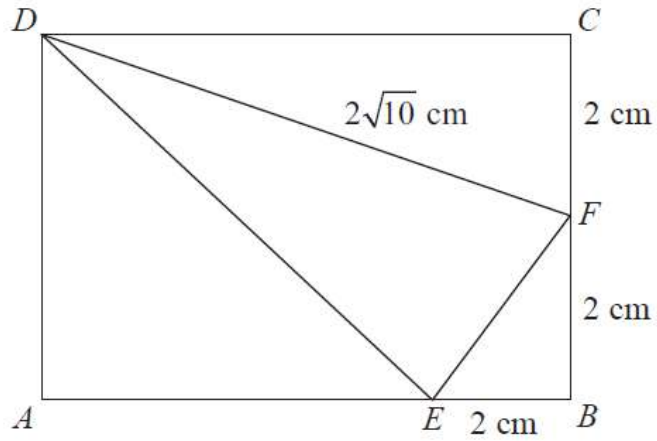


Diagram **NOT**
accurately drawn

Show that the area of triangle DEF is 8 cm^2 .
You must show all your working.

7 The diagram shows a triangle DEF inside a rectangle $ABCD$.

$$a = \sqrt{8} + 2$$

$$b = \sqrt{8} - 2$$

$$T = a^2 - b^2$$

Work out the value of T .

Give your answer in the form $c\sqrt{2}$ where c is an integer.

.....

Answers

1 a -1

b $9 - \sqrt{3}$

c $10\sqrt{5} - 7$

d $26 - 4\sqrt{2}$

2 $x - y$

3 18

4 $1 + 2\sqrt{2}$

5 $\frac{1}{4} - \frac{\sqrt{6}}{12}$

Method:

$$\frac{1}{2} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} \text{ or } \frac{1}{2} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{3}$$

$$\frac{1}{2} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{2}}{2} - \frac{1}{2} \times \frac{\sqrt{2}}{2} \times \frac{\sqrt{3}}{3}$$

$$\frac{1}{4} - \frac{\sqrt{6}}{12} \text{ oe}$$

OR

$$(\text{BC} =) \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{3}$$

$$\frac{1}{2} \times \left\{ \frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{3} \right\} \times \frac{\sqrt{2}}{2}$$

$$\frac{1}{4} - \frac{\sqrt{6}}{12} \text{ oe}$$

6 8

Method:

$$(2\sqrt{10})^2 - 2^2 (= 36)$$

$$(\text{CD} =) 6$$

$$'6' \times 4 - \frac{1}{2} \times '6' \times 2 - \frac{1}{2} \times 2 \times 2 - \frac{1}{2} \times ('6' - 2) \times 4$$

7 $16\sqrt{2}$

Exercise 4: Factorising quadratics

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Example 1 Factorise $x^2 + 3x - 10$

$b = 3, ac = -10$ <p>So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$</p> $= x(x + 5) - 2(x + 5)$ $= (x + 5)(x - 2)$	<ol style="list-style-type: none"> 1 Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and -2) 2 Rewrite the b term ($3x$) using these two factors 3 Factorise the first two terms and the last two terms 4 $(x + 5)$ is a factor of both terms
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Example 2 Factorise $6x^2 - 11x - 10$

$b = -11, ac = -60$ <p>So</p> $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	<ol style="list-style-type: none"> 1 Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) 2 Rewrite the b term ($-11x$) using these two factors 3 Factorise the first two terms and the last two terms 4 $(2x - 5)$ is a factor of both terms
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Example 3 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	<p>This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$</p>
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Practice questions

1 Factorise

a $x^2 + 7x + 12$

c $x^2 - 11x + 30$

e $x^2 - 7x - 18$

g $x^2 - 3x - 40$

b $x^2 + 5x - 14$

d $x^2 - 5x - 24$

f $x^2 + x - 20$

h $x^2 + 3x - 28$

2 Factorise fully

a $y^2 - 100$

c $4x^2 - 81y^2$

b $36x^2 - 49y^2$

d $18a^2 - 200b^2c^2$

3 Factorise fully

a $2x^2 + x - 3$

c $2x^2 + 7x + 3$

e $10x^2 + 21x + 9$

b $6x^2 + 17x + 5$

d $9x^2 - 15x + 4$

f $12x^2 - 38x + 20$

Answers

1 a $(x + 3)(x + 4)$

c $(x - 5)(x - 6)$

e $(x - 9)(x + 2)$

g $(x - 8)(x + 5)$

b $(x + 7)(x - 2)$

d $(x - 8)(x + 3)$

f $(x + 5)(x - 4)$

h $(x + 7)(x - 4)$

2 a $(y - 10)(y + 10)$

c $(2x - 9y)(2x + 9y)$

b $(6x - 7y)(6x + 7y)$

d $2(3a - 10bc)(3a + 10bc)$

3 a $(x - 1)(2x + 3)$

c $(2x + 1)(x + 3)$

e $(5x + 3)(2x + 3)$

b $(3x + 1)(2x + 5)$

d $(3x - 1)(3x - 4)$

f $2(3x - 2)(2x - 5)$

Exercise 5: Solving quadratic equations

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- A quadratic equation is an equation in the form $ax^2 + bx + c = 0$ where $a \neq 0$.
- To factorise a quadratic equation find two numbers whose sum is b and whose products is ac .
- When the product of two numbers is 0, then at least one of the numbers must be 0.
- If a quadratic can be solved it will have two solutions (these may be equal).

Example 1 Solve $5x^2 = 15x$

$5x^2 = 15x$ $5x^2 - 15x = 0$ $5x(x - 3) = 0$ <p>So $5x = 0$ or $(x - 3) = 0$</p> <p>Therefore $x = 0$ or $x = 3$</p>	<ol style="list-style-type: none"> 1 Rearrange the equation so that all of the terms are on one side of the equation and it is equal to zero. Do not divide both sides by x as this would lose the solution $x = 0$. 2 Factorise the quadratic equation. $5x$ is a common factor. 3 When two values multiply to make zero, at least one of the values must be zero. 4 Solve these two equations.
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Example 2 Solve $x^2 + 7x + 12 = 0$

$x^2 + 7x + 12 = 0$ $b = 7, ac = 12$ $x^2 + 4x + 3x + 12 = 0$ $x(x + 4) + 3(x + 4) = 0$ $(x + 4)(x + 3) = 0$ <p>So $(x + 4) = 0$ or $(x + 3) = 0$</p> <p>Therefore $x = -4$ or $x = -3$</p>	<ol style="list-style-type: none"> 1 Factorise the quadratic equation. Work out the two factors of $ac = 12$ which add to give you $b = 7$. (4 and 3) 2 Rewrite the b term ($7x$) using these two factors. 3 Factorise the first two terms and the last two terms. 4 $(x + 4)$ is a factor of both terms. 5 When two values multiply to make zero, at least one of the values must be zero. 6 Solve these two equations.
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Example 3 Solve $9x^2 - 16 = 0$

$9x^2 - 16 = 0$ $(3x + 4)(3x - 4) = 0$ <p>So $(3x + 4) = 0$ or $(3x - 4) = 0$</p> $x = -\frac{4}{3} \text{ or } x = \frac{4}{3}$	<ol style="list-style-type: none"> 1 Factorise the quadratic equation. This is the difference of two squares as the two terms are $(3x)^2$ and $(4)^2$. 2 When two values multiply to make zero, at least one of the values must be zero. 3 Solve these two equations.
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Example 4 Solve $2x^2 - 5x - 12 = 0$

$b = -5, ac = -24$ <p>So $2x^2 - 8x + 3x - 12 = 0$</p> $2x(x - 4) + 3(x - 4) = 0$ $(x - 4)(2x + 3) = 0$ <p>So $(x - 4) = 0$ or $(2x + 3) = 0$</p> $x = 4 \text{ or } x = -\frac{3}{2}$	<ol style="list-style-type: none"> 1 Factorise the quadratic equation. Work out the two factors of $ac = -24$ which add to give you $b = -5$. (-8 and 3) 2 Rewrite the b term ($-5x$) using these two factors. 3 Factorise the first two terms and the last two terms. 4 $(x - 4)$ is a factor of both terms. 5 When two values multiply to make zero, at least one of the values must be zero. 6 Solve these two equations.
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Practice questions

1 Solve

a $6x^2 + 4x = 0$

c $x^2 + 7x + 10 = 0$

e $x^2 - 3x - 4 = 0$

g $x^2 - 10x + 24 = 0$

i $x^2 + 3x - 28 = 0$

k $2x^2 - 7x - 4 = 0$

b $28x^2 - 21x = 0$

d $x^2 - 5x + 6 = 0$

f $x^2 + 3x - 10 = 0$

h $x^2 - 36 = 0$

j $x^2 - 6x + 9 = 0$

l $3x^2 - 13x - 10 = 0$

2 Solve

a $x^2 - 3x = 10$

c $x^2 + 5x = 24$

e $x(x + 2) = 2x + 25$

g $x(3x + 1) = x^2 + 15$

b $x^2 - 3 = 2x$

d $x^2 - 42 = x$

f $x^2 - 30 = 3x - 2$

h $3x(x - 1) = 2(x + 1)$

Hint

Get all terms onto one side of the equation.

Answers

- 1**
- | | | | |
|----------|-------------------------------|----------|-------------------------------|
| a | $x = 0$ or $x = -\frac{2}{3}$ | b | $x = 0$ or $x = \frac{3}{4}$ |
| c | $x = -5$ or $x = -2$ | d | $x = 2$ or $x = 3$ |
| e | $x = -1$ or $x = 4$ | f | $x = -5$ or $x = 2$ |
| g | $x = 4$ or $x = 6$ | h | $x = -6$ or $x = 6$ |
| i | $x = -7$ or $x = 4$ | j | $x = 3$ |
| k | $x = -\frac{1}{2}$ or $x = 4$ | l | $x = -\frac{2}{3}$ or $x = 5$ |
- 2**
- | | | | |
|----------|--------------------------------|----------|-------------------------------|
| a | $x = -2$ or $x = 5$ | b | $x = -1$ or $x = 3$ |
| c | $x = -8$ or $x = 3$ | d | $x = -6$ or $x = 7$ |
| e | $x = -5$ or $x = 5$ | f | $x = -4$ or $x = 7$ |
| g | $x = -3$ or $x = 2\frac{1}{2}$ | h | $x = -\frac{1}{3}$ or $x = 2$ |

Exercise 6: Simplifying algebraic fractions

A LEVEL LINKS

Scheme of work: 1a. Algebraic expressions – basic algebraic manipulation, indices and surds

Example 1 Factorise $x^2 + 3x - 10$

$b = 3, ac = -10$ <p>So $x^2 + 3x - 10 = x^2 + 5x - 2x - 10$</p> $= x(x + 5) - 2(x + 5)$ $= (x + 5)(x - 2)$	<ol style="list-style-type: none"> 1 Work out the two factors of $ac = -10$ which add to give $b = 3$ (5 and -2) 2 Rewrite the b term ($3x$) using these two factors 3 Factorise the first two terms and the last two terms 4 $(x + 5)$ is a factor of both terms
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Example 2 Factorise $6x^2 - 11x - 10$

$b = -11, ac = -60$ <p>So</p> $6x^2 - 11x - 10 = 6x^2 - 15x + 4x - 10$ $= 3x(2x - 5) + 2(2x - 5)$ $= (2x - 5)(3x + 2)$	<ol style="list-style-type: none"> 1 Work out the two factors of $ac = -60$ which add to give $b = -11$ (-15 and 4) 2 Rewrite the b term ($-11x$) using these two factors 3 Factorise the first two terms and the last two terms 4 $(2x - 5)$ is a factor of both terms
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Example 3 Factorise $4x^2 - 25y^2$

$4x^2 - 25y^2 = (2x + 5y)(2x - 5y)$	<p>This is the difference of two squares as the two terms can be written as $(2x)^2$ and $(5y)^2$</p>
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Example 4 Simplify $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$

$\frac{x^2 - 4x - 21}{2x^2 + 9x + 9}$ <p>For the numerator: $b = -4, ac = -21$</p> <p>So $x^2 - 4x - 21 = x^2 - 7x + 3x - 21$ $= x(x - 7) + 3(x - 7)$ $= (x - 7)(x + 3)$</p> <p>For the denominator: $b = 9, ac = 18$</p> <p>So $2x^2 + 9x + 9 = 2x^2 + 6x + 3x + 9$ $= 2x(x + 3) + 3(x + 3)$ $= (x + 3)(2x + 3)$</p> <p>So $\frac{x^2 - 4x - 21}{2x^2 + 9x + 9} = \frac{(x - 7)(x + 3)}{(x + 3)(2x + 3)}$ $= \frac{x - 7}{2x + 3}$</p>	<ol style="list-style-type: none"> 1 Factorise the numerator and the denominator 2 Work out the two factors of $ac = -21$ which add to give $b = -4$ (-7 and 3) 3 Rewrite the b term ($-4x$) using these two factors 4 Factorise the first two terms and the last two terms 5 $(x - 7)$ is a factor of both terms 6 Work out the two factors of $ac = 18$ which add to give $b = 9$ (6 and 3) 7 Rewrite the b term ($9x$) using these two factors 8 Factorise the first two terms and the last two terms 9 $(x + 3)$ is a factor of both terms 10 $(x + 3)$ is a factor of both the numerator and denominator so cancels out as a value divided by itself is 1
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Practice questions

1 Simplify the algebraic fractions.

a $\frac{2x^2 + 4x}{x^2 - x}$

c $\frac{x^2 - 2x - 8}{x^2 - 4x}$

e $\frac{x^2 - x - 12}{x^2 - 4x}$

b $\frac{x^2 + 3x}{x^2 + 2x - 3}$

d $\frac{x^2 - 5x}{x^2 - 25}$

f $\frac{2x^2 + 14x}{2x^2 + 4x - 70}$

2 Simplify

a $\frac{9x^2 - 16}{3x^2 + 17x - 28}$

c $\frac{4 - 25x^2}{10x^2 - 11x - 6}$

b $\frac{2x^2 - 7x - 15}{3x^2 - 17x + 10}$

d $\frac{6x^2 - x - 1}{2x^2 + 7x - 4}$

Extend

3 Simplify $\sqrt{x^2 + 10x + 25}$

4 Simplify $\frac{(x+2)^2 + 3(x+2)^2}{x^2 - 4}$

Answers

1 a $\frac{2(x+2)}{x-1}$

c $\frac{x+2}{x}$

e $\frac{x+3}{x}$

b $\frac{x}{x-1}$

d $\frac{x}{x+5}$

f $\frac{x}{x-5}$

2 a $\frac{3x+4}{x+7}$

c $\frac{2-5x}{2x-3}$

b $\frac{2x+3}{3x-2}$

d $\frac{3x+1}{x+4}$

3 $(x+5)$

4 $\frac{4(x+2)}{x-2}$

Exercise 7: Complete the square

A LEVEL LINKS

Scheme of work: 1b. Quadratic functions – factorising, solving, graphs and the discriminants

Key points

- Completing the square lets you write a quadratic equation in the form $p(x + q)^2 + r$

Examples

Example 1 Complete the square for the expression $x^2 + 6x$

$x^2 + 6x$ $= \left(x + \frac{6}{2}\right)^2 - \left(\frac{6}{2}\right)^2$ $= (x + 3)^2 - 9$	<p>1 Write $x^2 + bx + c$ in the form $\left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c$</p> <p>2 Simplify.</p>
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Example 2 Complete the square for the expression $2x^2 - 7x$

$2x^2 - 7x$ $= 2\left(x^2 - \frac{7}{2}x\right)$ $= 2\left[\left(x - \frac{7}{4}\right)^2 - \left(\frac{7}{4}\right)^2\right]$ $= 2\left(x - \frac{7}{4}\right)^2 - \frac{49}{8}$	<p>1 Before completing the square write $ax^2 + bx + c$ in the form $a\left(x^2 + \frac{b}{a}x\right) + c$</p> <p>2 Now complete the square by writing $x^2 - \frac{7}{2}x$ in the form $\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2$</p> <p>3 Expand and Simplify</p>
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Practice questions

1 Complete the square for the following expressions:

a $x^2 + 8x$

b $x^2 - 10x$

c $x^2 - x$

d $3x^2 - 15x$

e $12x - 2x^2$

Answers

1 Solve by completing the square.

a $(x + 4)^2 - 16$

b $(x - 5)^2 - 25$

c $\left(x - \frac{1}{2}\right)^2 - \frac{1}{4}$

d $3\left(x - \frac{5}{2}\right)^2 - \frac{75}{4}$

e $-2(x - 3)^2 + 18$

Exercise 8: Solving linear simultaneous equations by elimination

A LEVEL LINKS

Scheme of work: 1c. Equations – quadratic/linear simultaneous

Key points

- Two equations are simultaneous when they are both true at the same time.
- Solving simultaneous linear equations in two unknowns involves finding the value of each unknown which works for both equations.
- Make sure that the coefficient of one of the unknowns is the same in both equations.
- Eliminate this equal unknown by either subtracting or adding the two equations.

Example 1 Solve the simultaneous equations $3x + y = 5$ and $x + y = 1$

$\begin{array}{r} 3x + y = 5 \\ - \quad x + y = 1 \\ \hline 2x \quad = 4 \\ \text{So } x = 2 \end{array}$ <p>Using $x + y = 1$ $2 + y = 1$ So $y = -1$</p> <p>Check: equation 1: $3 \times 2 + (-1) = 5$ YES equation 2: $2 + (-1) = 1$ YES</p>	<ol style="list-style-type: none"> 1 Subtract the second equation from the first equation to eliminate the y term. 2 To find the value of y, substitute $x = 2$ into one of the original equations. 3 Substitute the values of x and y into both equations to check your answers.
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Example 2 Solve $x + 2y = 13$ and $5x - 2y = 5$ simultaneously.

$\begin{array}{r} x + 2y = 13 \\ + 5x - 2y = 5 \\ \hline 6x \quad = 18 \\ \text{So } x = 3 \end{array}$ <p>Using $x + 2y = 13$ $3 + 2y = 13$ So $y = 5$</p> <p>Check: equation 1: $3 + 2 \times 5 = 13$ YES equation 2: $5 \times 3 - 2 \times 5 = 5$ YES</p>	<p>1 Add the two equations together to eliminate the y term.</p> <p>2 To find the value of y, substitute $x = 3$ into one of the original equations.</p> <p>3 Substitute the values of x and y into both equations to check your answers.</p>
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Practice questions

Solve these simultaneous equations.

1 $4x + y = 8$
 $x + y = 5$

2 $3x + y = 7$
 $3x + 2y = 5$

3 $4x + y = 3$
 $3x - y = 11$

4 $3x + 4y = 7$
 $x - 4y = 5$

5 $2x + y = 11$
 $x - 3y = 9$

6 $2x + 3y = 11$
 $3x + 2y = 4$

7 $4x + y = 25$
 $x - 3y = 16$

Answers

1 $x = 1, y = 4$

2 $x = 3, y = -2$

3 $x = 2, y = -5$

4 $x = 3, y = -\frac{1}{2}$

5 $x = 6, y = -1$

6 $x = -2, y = 5$

7 $x = 7, y = -3$